Homework #1

November 13, 2009

The assignments are due Friday, November 20 at 11AM in class. All questions have equal weight. Have fun!

**Exercise 1**

1. For each point in the following model give a modal formula that is only true at it. Hint: use $\top$ and $\bot$.

```
1 ---- 2
\|    \|
4 ---- 3
```

2. Which states can you uniquely characterize by modal formulas in the next model? Give the formulas and explain your answer.

```
1 ---- 2
\|    \|
4 ---- 3
```

**Exercise 2**

1. Determine in which states of the following model the modal formula $\lozenge \square \lozenge p$ is true.

2. Give a complete game tree for the evaluation game for $\lozenge \square \lozenge p$ in this model starting in state 1.

```
1:p ---- 2:p
\|      \|
\|      \|
4 ---- 3
```


Exercise 3

1. Draw a bisimulation between the following two models, connecting points A and D. Check that it satisfies the definition.

```
A ─── B:p
  ^  
  |   
  v  
C ─── D
  ^  
  |   
  v  
E:p
```

2. Show that there is no bisimulation between the points A and D in the next two models by giving a modal formula true in only one of them.

```
A ─── B:p
  ^  
  |   
  v  
C ─── D ─── E:p
  ^   ^
  |   |
  v   v
```

Exercise 4

1. Prove that:
   
   (a) formula ♦(P → Q) → (□P → ♦Q) is valid;
   
   (b) formula □(□P → Q) ∨ □(□Q → P) is not valid.

2. Compute the modal depth for both formulas.

Exercise 5  Prove the following. Let f be an isomorphism between models M = (W,R,V) and M′ = (W′,R′,V′). Then for all basic modal formulas φ, and all points w in M, we have that:

\[ M, w \models φ \iff M′, f(w) \models φ \]