Homework #2

November 20, 2009

The assignments are due Friday, November 27 at 11AM in class. All questions have equal weight. Good luck!

**Exercise 1** Recall exercise 3 from the previous homework.

1. Give a winning strategy for Duplicator in the bisimulation game played on the following models from the match between points A and D.

```
A -- B:p ------------------------------ C -- D
   \        \                      /        /
     \      \                    /      /
       \    \                  /    /
          \  \              /  /
            \ \            / /
              \ \          /  
                \ \        /   
                  \ \      /    
                    \ \    /     
                      \ \ /      
                        \ /       
                          \       
                            \    
                              \   
                                \  
                                  \ 
                                    E:p
```

2. Give a winning strategy for Spoiler in the bisimulation game played on the following models from the match between points A and D. How many moves does Spoiler need?

```
A ---- B:p
    \   /   
      \ /    
        \     
          \    
            \   
              \  
                \ 
                  \ 
                    \ 
                      C ------ D ---- E:p
```

**Exercise 2** Consider the following new operator □⁻ defined on relational structures $M = (W, R, V)$ in the following way:

$M, w \models □⁻ \varphi$ iff for all $v \in W$, if $vRw$ then $M, v \models \varphi$

Prove that □⁻ is not definable in the basic modal language.

**Exercise 3**

1. Prove the following formula in the system K: $\Box(\varphi \land \psi) \iff (\Box \varphi \land \Box \psi)$.

2. Prove the formula $\Box \Diamond \Box \Diamond \varphi \iff \Box \Diamond \varphi$ in the logic S4.

**Exercise 4** Prove soundness theorem for K:

If $\vdash_K \varphi$ then $\varphi$ holds on all frames.

**Exercise 5** We say that frame $F = (W, R)$ is Euclidean if the relation $R$ is Euclidean (for all $x, y, z \in W$ if $xRy$ and $xRz$ then $yRz$). Prove that $F$ is Euclidean iff $F \models \Diamond \varphi \implies \Box \Diamond \varphi$. 