Computational complexity of some Ramsey quantifiers in finite models

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The problem of computational complexity of semantics for some natural language constructions – considered in [M. Mostowski, D. Wojtyniak 2004] – motivates an interest in complexity of Ramsey quantifiers in finite models. In general a sentence with a Ramsey quantifier R of the following form $\mathsf{R}x, yH(x, y)$ is interpreted as $\exists A(A$ is big relatively to the universe $\wedge A^2 \subseteq H)$. In the paper cited the problem of the complexity of the Hintikka sentence is reduced to the problem of computational complexity of the Ramsey quantifier for which the phrase "A is big relatively to the universe" is interpreted as containing at least one representative of each equivalence class, for some given equvalence relation.

In this work we consider quantifiers R_f, for which "A is big relatively to the universe" means "$card(A) > f(n)$, where n is the size of the universe". Following [Blass, Gurevich 1986] we call R mighty if $\mathsf{R}x, yH(x, y)$ defines NP–complete class of finite models. Similarly we say that R_f is NP–hard if the corresponding class is NP–hard. We prove the following theorems:

Theorem 1 *Let $f(n)$ be the integral part of $r \times n$, for some rational r such that $0 < r < 1$. Thus R_f is mighty.*

Theorem 2 *Let f be such that $\lim_{n\to\infty} f(n)/n = a$ exists and $0 < a < 1$. Then R_f is NP–hard.*

Theorem 3 *If f satisfies the assumptions of the previous theorem and f is $PTIME$ computable then R_f is mighty.*

References

[Blass, Gurevich 1986] A. Blass and Y. Gurevich, *Henkin Quantifiers and Complete Problems*, **Annals of Pure and Applied Logic** Vol. 32 (1986), pp. 1 – 16.

[M. Mostowski, D. Wojtyniak 2004] M. MOSTOWSKI, D. WOJTYNIAK *Computational Complexity of the Semantics of Some Natural Language Constructions*, **Annals of Pure and Applied Logic** Vol. 127 (2004), 1 – 3, pp. 219 – 227.