

# MOST OF THE NPS AREN'T NP-HARD

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# OUTLINE

- 1 INTRODUCTION
- 2 SIMPLE QUANTIFIERS
- 3 COMPLEX QUANTIFIERS
  - Some Happen to Be Intractable
  - But Most of Them Are Tractable
- 4 LANGUAGE COMPLEXITY GAME

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# QUANTIFIERS IN LANGUAGE

- Quantifiers occur whenever we speak.
- They influence language expressivity.
- GQT studies their meaning.
- Interface of linguistics, mathematics and philosophy.

# WHICH GQS ARE REALIZED IN NL?

- Topic neutrality (isomorphism);
- Domain independence;
- Conservativity;
- Continuity;
- FIN;
- ...

# DESCRIPTIVE COMPLEXITY OF QUANTIFIERS

- How much logic is needed to formalize NL?
- Hierarchy of various fragments of NL.
  - “Some”, “At least 5”, and “More than 7” are FO.
  - But “An even number of” and “Most” are not.

# COMPUTATIONAL COMPLEXITY OF QUANTIFIERS

- How much resources is needed for processing?
- W.r.t. to model size.
- Restriction to finite models.
- How difficult is it to understand quantifiers?

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## NL DETERMINERS...

- ...mostly correspond to GQs of type (1,1).

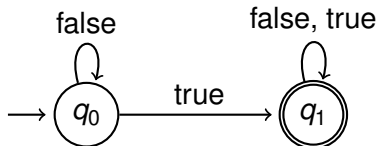
### DEFINITION

A monadic generalized quantifier of type (1,1) is a class  $Q$  of structures of the form  $M = (U, A_1, A_2)$ , where  $A_1, A_2 \subseteq U$ . Additionally,  $Q$  is closed under isomorphism.

- They can be easily computed.

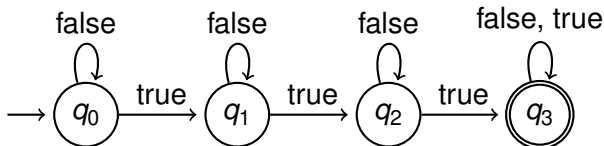
## EXAMPLE 1: ARISTOTELIAN QUANTIFIERS

- Some sentences in my thesis are true.



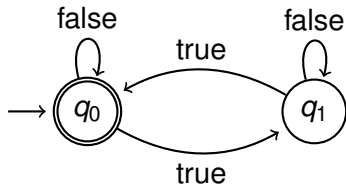
## EXAMPLE 2: CARDINAL QUANTIFIERS

- At least 3 sentences in my thesis are true.



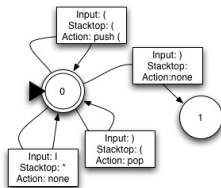
## EXAMPLE 3: PARITY QUANTIFIERS

- An even number of sentences in my thesis is true.



## EXAMPLE 4: PROPORTIONAL QUANTIFIERS

- “Most of the sentences in my thesis are true.”
- Not computable by finite-automata.
- We need working memory.
- Simple push-down automata will do.



## BUT...ARE THEY EASY TO UNDERSTAND?

- And... does it say anything about comprehension?
- Empirical evidence converges to “YES”.

## BRAIN ACTIVITY DURING COMPREHENSION

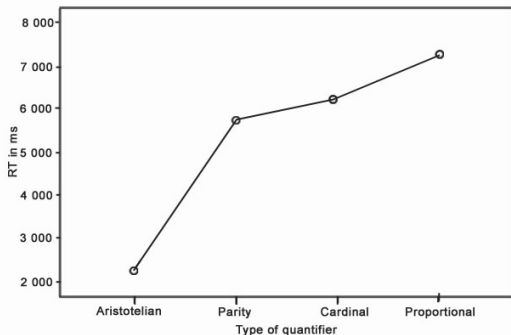
- All quantifiers are associated with numerosity;
- Proportional activate working-memory capacity;
- But FO-quantifiers don't!

## ACCURACY AS DIFFICULTY MEASURE

| Quantifier group | Examples                       | Percent |
|------------------|--------------------------------|---------|
| Aristotelian FO  | all, some                      | 99      |
| Parity           | odd, even                      | 91      |
| Cardinal FO      | less than 8, more than 7       | 92      |
| Proportional     | less than half, more than half | 85      |

The percentage of correct answers for different types of quantifiers.

# REACTION TIME AS DIFFICULTY MEASURE



Average reaction times in each type of quantifiers

# WHAT NEXT?

- What about more complex quantifiers?
- What about the border tractable/intractable?

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# POLYADIC GQS

## DEFINITION

A *generalized quantifier*  $Q$  of type  $t = (n_1, \dots, n_k)$  is a functor assigning to every set  $M$  a  $k$ -ary relation  $Q_M$  between relations on  $M$  such that if  $(R_1, \dots, R_k) \in Q_M$  then  $R_i$  is an  $n_i$ -ary relation on  $M$ , for  $i = 1, \dots, k$ . Additionally,  $Q$  is preserved by bijections.

## DEFINITION

If for all  $i$  the relation  $R_i$  is unary, i.e. it denotes a subset of the universe, then we say that the quantifier is *monadic*. Otherwise, it is *polyadic*.

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# POSSIBLY BRANCHING SENTENCES

- 1 Most villagers and most townsmen hate each other.
- 2 One third of villagers and half of townsmen hate each other.
- 3 5 villagers and 7 townsmen hate each other.

## BRANCHING READING

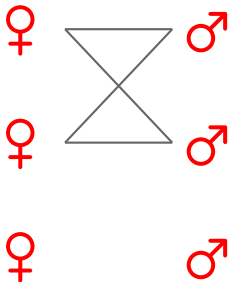
Most girls and most boys hate each other.

BRANCHING:  $\text{most } x : G(x)$   
 $\text{most } y : B(y) \quad H(x, y).$

$\exists A \exists A' [\text{most}(G, A) \wedge \text{most}(B, A') \wedge \forall x \in A \forall y \in A' H(x, y)].$

## ILLUSTRATION

Most girls and most boys hate each other.



# BRANCHING READINGS ARE INTRACTABLE

## THEOREM

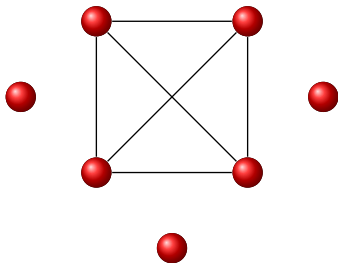
*Proportional branching sentences are NP-complete.*

# POTENTIALLY STRONG RECIPROCAL SENTENCES

- 1 Andi, Jarmo and Jakub laughed at **one another**.
- 2 15 men are hitting **one another**.
- 3 Most of the PMs refer to **each other**.

# STRONG READING

Most of the PMs refer to each other.



## STRONG RECIPROCAL LIFT

Let  $Q$  be a monadic monotone increasing quantifier.

### DEFINITION

$\text{Ram}_S(Q)(A, R) \iff$

$$\exists X \subseteq A [Q(X) \wedge \forall x, y \in X (x \neq y \Rightarrow R(x, y))].$$

### EXAMPLE

- Most of the PMs refer to each other indirectly.
- $\text{Ram}_S(\text{most})[\text{PMs}, \text{Refer}]$ .

# STRONG RECIPROCITY IS INTRACTABLE

## THEOREM

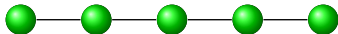
*Model-checking for strong reciprocal sentences with proportional quantifiers is NP-complete.*

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## INTERMEDIATE READING

Most Boston pitchers sat alongside each other.



## INTERMEDIATE RECIPROCAL LIFT

### DEFINITION

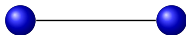
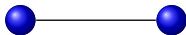
$$\text{Ram}_1(Q)(A, R) \iff \exists X \subseteq A [Q(X) \wedge \forall x, y \in X \\ (x \neq y \Rightarrow \exists \text{ sequence } z_1, \dots, z_\ell \in X \text{ such that} \\ (z_1 = x \wedge R(z_1, z_2) \wedge \dots \wedge R(z_{\ell-1}, z_\ell) \wedge z_\ell = y))].$$

### EXAMPLE

- Most Boston pitchers sat alongside each other.
- $\text{Ram}_1(\text{most})[\text{Pitcher}, \text{Sit}]$ .

# WEAK READING

Some pirates were staring at each other in surprise.



## WEAK RECIPROCAL LIFT

### DEFINITION

$\text{Ram}_W(Q)(A, R) \iff$

$$\exists X \subseteq A [Q(X) \wedge \forall x \in X \exists y \in X (x \neq y \wedge R(x, y))].$$

### EXAMPLE

- Some pirates were staring at each other in surprise.
- $\text{Ram}_W(\text{some})[\text{Pirate}, \text{Staring}]$ .

# COMPLEXITY DICHOTOMY

## THEOREM

*If  $Q$  is PTIME, then also  $\text{Ram}_I(Q)$  and  $\text{Ram}_W(Q)$  are in PTIME.*

# ITERATION

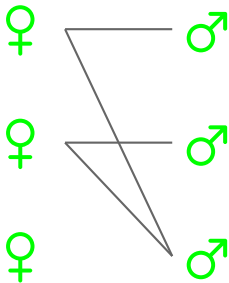
- 1 Most logicians criticized some papers.
- 2  $\text{It}(\text{most, some})[\text{Logicians, Papers, Criticized}]$ .

## DEFINITION

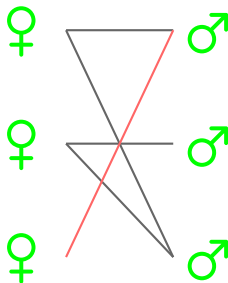
$\text{It}(Q, Q')[A, B, R] \iff Q[A, \{a \mid Q'(B, R_{(a)})\}]$ , where  
 $R_{(a)} = \{b \mid R(a, b)\}$ .

# ITERATION OR BRANCHING?

Most girls and most boys hate each other.



# RATHER TWO-SIDE-ITERATION



# CUMULATION

- 1 Eighty professors taught sixty courses at ESSLLI'08.

## DEFINITION

$\text{Cum}(Q, Q')[A, B, R] \iff$

$$\text{It}(Q, \text{some})[A, B, R] \wedge \text{It}(Q', \text{some})[B, A, R^{-1}]$$

## BASIC OPERATIONS ARE TRACTABLE

### THEOREM

*Let  $Q$  and  $Q'$  be generalized quantifiers computable in PTIME with respect to the size of a universe. Then the quantifiers: (1)  $\neg Q$ ; (2)  $Q\neg$ ; (3)  $Q \wedge Q'$ ; (4)  $\text{It}(Q, Q')$ ; (5)  $\text{Cum}(Q, Q')$ ; (6)  $\text{Res}(Q)$  are PTIME computable.*

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# COMPUTATIONAL APPROACH TO COGNITION

- Cognitive task is a computational task.
- *CT* : Initial state of the world  $\longrightarrow$  Desired final state.
- Dominant view in cognitive and experimental psychology.

# COMPREHENSION TASK

- Model-checking is a part of comprehension.
- Input:  $\mathbb{M}$ ,  $\varphi$ . Output:  $\varphi^{\mathbb{M}}$ .
- Algorithmic theory of meaning.
- Psychological motivations.

# MARR'S LEVELS OF EXPLANATION

- 1 Computational level;
- 2 Algorithmic level;
- 3 Implementation level.

# COMPUTATIONAL BOUNDS ON COGNITION

- Computational complexity of a CT — level 1.5;
- Psychological version of the Church-Turing Thesis.

# TRACTABLE COGNITION

## HYPOTHESIS

*Human cognitive (linguistic) capacities are constrained by polynomial time computability.*

## HYPOTHESIS

*This is why most NPs aren't NP-complete.*