Quantifiers, Automata and Language Comprehension

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Abstract

- Comprehension of simple quantifiers in natural language.
- Computational model posited by many logicians.
- Linking computational complexity and cognitive science.
- Comparing RT needed for understanding:
  - FA-quantifiers vs. PDA-quantifiers;
  - Aristotelian quantifiers vs. cardinal quantifiers;
  - Parity quantifiers;
  - PDA-quantifiers over ordered and unordered universes.

Outline

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Computability and cognition

- A cognitive task is a computational task.
- Marr's levels: computational, algorithmic, neurological.
- Today computational restrictions are taken seriously.
  - Tsotsos, Analyzing vision at the complexity level, 1990.
  - Frixione, Tractable competence, 2001.
  - van Rooij, The tractable cognition thesis, 2008.
- But no empirical links, too abstract considerations.

MEANING AS ALGORITHM

- Ability of understanding sentences.
- Capacity of recognizing their truth-values.
- Fregean tradition.
- Meaning is a procedure for finding extension in a model.
- Adopted often with psychological motivations.
  - Suppes, Variable-free semantics with remark on procedural extensions, 1982.
  - Lambalgen & Hamm, The Proper Treatment of Events, 2005.

Previous investigations

- Quantifiers widely treated in cognitive psychology.
- Brain activity during the comprehension of:
  - FO-quantifiers vs. higher-order quantifiers;
  - All quantifies are associated with numerosity;
  - Higher-order activate working-memory capacity;
  - Computational differences missed;
  - “Even” is higher-order but FA-computable.

McMillan et al., “Neural basis for generalized quantifiers comprehension”, 2005.
Szymanik, “A Note on a Neuroimaging Study of Natural Language Quantifiers Comprehension”, 2007.

Outline

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Motivations

Quantifiers and Automata

- Generalized Quantifiers
- Automata for Quantifiers

The Experiment

- Comparing Quantifiers
- Quantifiers and Ordering

Conclusions and Perspectives

Simple quantifier sentences

- Every poet has low self-esteem.
- Some dean danced nude on the table.
- At least 3 grad students prepared presentations.
- An even number of the students saw a ghost.
- Most of the students think they are smart.
- Less than half of the students received good marks.

Lindström definition

Definition

A monadic generalized quantifier of type (1,1) is a class Q of structures of the form $M = (U, A_1, A_2)$, where $A_1, A_2 \subseteq U$. Additionally, Q is closed under isomorphism.

A few examples

- $K_{Some} = \{(U, A, B) : A, B \subseteq U \wedge A \cap B \neq \emptyset\}$
- $K_{All} = \{(U, A, B) : A, B \subseteq U \wedge A \subseteq B\}$
- $K_{Exactly\ m} = \{(U, A, B) : A, B \subseteq U \wedge \text{card}(A \cap B) = m\}$
- $K_{D_n} = \{(U, A, B) : A, B \subseteq U \wedge \text{card}(A \cap B) = k \times n\}$
- $K_{Most} = \{(U, A, B) : \text{card}(A \cap B) > \text{card}(A - B)\}$

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A FEW EXAMPLES

- $K_{Some} = \{(U, A, B) : A, B \subseteq U \wedge A \cap B \neq \emptyset\}$
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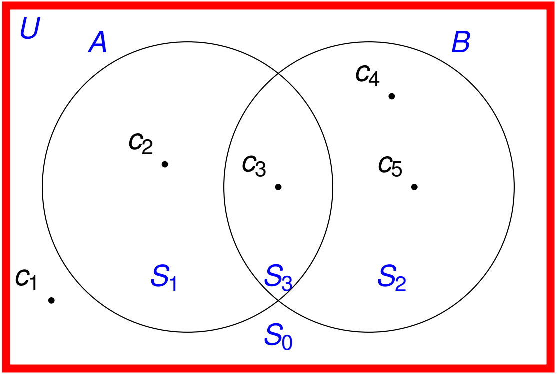
OUTLINE

How do we encode models?

- Restriction to finite models of the form $M = (U, A, B)$.
- List of all elements of the model: $c_1, \ldots, c_5$.
- Labeling every element with one of the letters: $a_{\bar{A}\bar{B}}$, $a_{A\bar{B}}$, $a_{\bar{A}B}$, a_{AB}, according to constituents it belongs to.
- Result: the word $\alpha_M = a_{\bar{A}\bar{B}}a_{A\bar{B}}a_{AB}a_{\bar{A}B}a_{\bar{A}B}$.
- α_M describes the model in which: $c_1 \in \bar{A}\bar{B}, c_2 \in A\bar{B} c_3 \in AB, c_4 \in \bar{A}B, c_5 \in \bar{A}B$.
- The class Q is represented by the set of words describing all elements of the class.

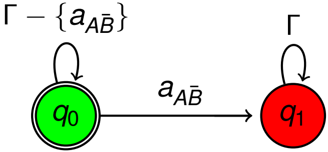
Illustration



This model is uniquely described by $\alpha_M = a_{\bar{A}\bar{B}} a_{A\bar{B}} a_{AB} a_{\bar{A}B} a_{\bar{A}B}$

Aristotelian quantifiers

"all", "some", "no", and "not all"

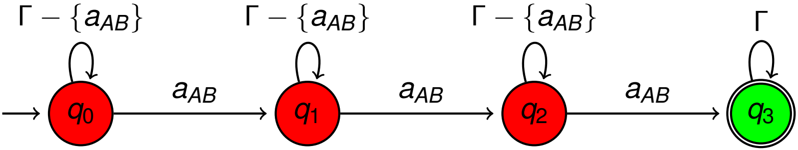


Finite automaton recognizing L_{Every}

$$L_{\text{Every}} = \{\alpha \in \Gamma^* : \#a_{A\bar{B}}(\alpha) = 0\}$$

CARDINAL QUANTIFIERS

E.g. “at least 3”, “at most 7”, and “between 8 and 11”

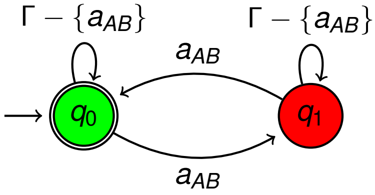


Finite automaton recognizing $L_{\text{At least three}}$

$$L_{\text{At least three}} = \{\alpha \in \Gamma^* : \#a_{AB}(\alpha) \geq 3\}$$

Parity quantifiers

E.g. “an even number”, “an odd number”



Finite automaton recognizing L_{Even}

$$L_{\text{Even}} = \{\alpha \in \Gamma^* : \#a_{AB}(\alpha) \text{ is even}\}$$

Proportional quantifiers

- E.g. "most", "less than half".
- Most *As are B* iff $\text{card}(A \cap B) > \text{card}(A - B)$.
- $L_{\text{Most}} = \{\alpha \in \Gamma^* : \#a_{AB}(\alpha) > \#a_{A\bar{B}}(\alpha)\}$.
- There is no finite automaton recognizing this language.
- We need internal memory.
- A push-down automata will do.

What does it mean that class of monadic quantifiers is recognized by class of devices?

Definition

Let $\mathcal{D}$ be a class of recognizing devices,
Ω a class of monadic quantifiers.
We say that $\mathcal{D}$ accepts Ω if and only if
for every monadic quantifier Q:

$$\mathsf{Q} \in \Omega \iff \text{there is device } A \in \mathcal{D}(A \textit{ accepts } L_{\mathsf{Q}}).$$

IN GENERAL

Definability	Examples	Recognized by
FO	"all" "at least 3"	acyclic FA
FO(D_n)	"an even number"	FA
PrA	"most", "less than half"	PDA

Quantifiers, definability, and complexity of automata

van Benthem, Essays in logical semantics, 1986.

Mostowski, Computational semantics for monadic quantifiers, 1998.

Outline

Generalities

- Joint work with M. Zajenkowski (University of Warsaw).
- 1st: RT in the comprehension of different quantifiers.
- 2nd: engagement of working-memory capacity.

Outline

General idea

- Compare RT wrt the following classes of quantifiers:
  - recognized by acyclic FA (first-order);
  - not first-order recognized by FA (divisibility);
  - recognized by PDA but not FA.
- Additionally:
  - Aristotelian vs. cardinal quantifiers of higher rank.

PREDICTIONS

- RT will increase along with the computational resources.
- Aristotelian qua. < parity qua. < proportional qua.
- Aristotelian qua. < cardinal qua. of high rank.
- Parity qua. < cardinal qua. of high rank.

PARTICIPANTS

- 40 native Polish-speaking adults.
- Volunteers: undergraduates from the University of Warsaw.
- 19 male and 21 female participants.
- The mean age: 21.42 years (SD = 3.22).
- Each participant tested individually.
- A small financial reward for participation in the study.

Materials

80 grammatically simple propositions in Polish, like:

1. Some cars are red.
2. More than 7 cars blue.
3. An even number of cars is yellow.
4. Less than half of the cars are black.

MATERIALS CONTINUED

Most of the cars are yellow.

An example of a stimulus used in the first study

Procedure

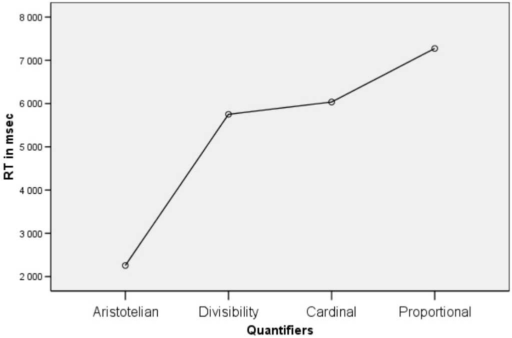
- 8 different quantifiers divided into four groups.
  - “all” and “some”;
  - “odd” and “even”;
  - “less than 8” and “more than 7”;
  - “less than half” and “more than half”.
- Each quantifier was presented in 10 trials.
- The sentence true in the picture in half of the trials.
- Quantity of target items near the criterion of validation.
- Practice session followed by the experimental session.
- Each quantifier problem was given one 15.5 s event.
- Subjects were asked to decide the truth-value.

Analysis of accuracy

Quantifier group	Examples	Percent
Aristotelian FO	all, some	99
Divisibility	odd, even	91
Cardinal FO	less than 8, more than 7	92
Proportional	less than half, more than half	85

The percentage of correct answers

Comparison of reaction times



Average reaction times in each type of quantifiers

To sum up

- Increase in RT was determined by the quantifier type.
- Pairwise comparisons: all four types of quantifiers differed significantly from one another.
- The mean reaction time increased as follows: Aristotelian, divisibility, cardinal, proportional.

Outline

GENERAL IDEA

- Investigating the role of working-memory capacity.
- The ordering as an additional independent variable.
- For example, consider the following sentence:
  "Most As are B."
- Universe ordered in pairs (a, b) such that $a \in A$, $b \in B$.

PREDICTIONS

- Given “good” ordering WM capacity is not needed.
- Ordering simplifies the problem = decrease in RT.

Participants

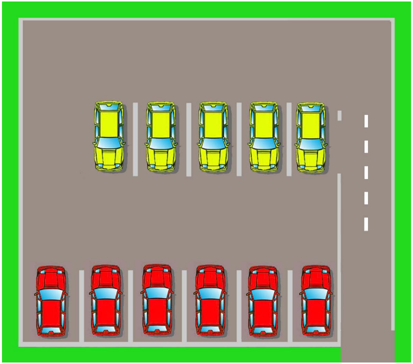
- 30 native Polish-speaking adults.
- Undergraduates from two Warsaw universities.
- 12 male and 18 female participants.
- The mean age: 23.4 years (SD = 2.51).
- Each subject tested individually.

Materials and procedure

- 16 grammatically simple propositions in Polish.
- E.g. “More than half of the cars are blue”.
- A car park with 11 cars.
- 2 quantifiers: “less than half” and “more than half”.
- Presented to each subject in 8 trials.
- Each type of sentence true in half of the trials.
- 4 ordered and 4 unordered pictures.
- The rest of the procedure the same as before.

EXAMPLE OF AN ORDERED TASK

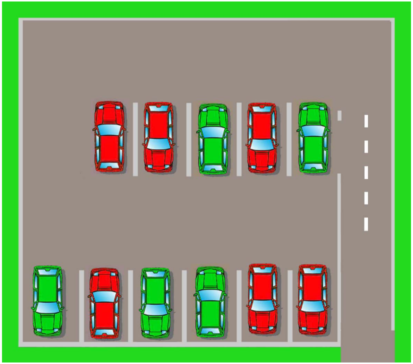
Most of the cars are red.



A case when cars are ordered

Example of an unordered task

Most of the cars are green.



A case when cars are distributed randomly

Results

- Higher accuracy of judgments for ordered universes (89%);
- Than for unordered (79%).
- Proportional quantifiers over randomized universes (M=6185.93; SD=1759.09);
- Over ordered models (M=4239.00; SD=1578.26);

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- Motivations
- Quantifiers and Automata
  - Generalized Quantifiers
  - Automata for Quantifiers
- The Experiment
  - Comparing Quantifiers
  - Quantifiers and Ordering
- **Conclusions and Perspectives**

Conclusions

- Plausibility of the model.
- Aristotelian easier than parity:
  loops influence the complexity of cognitive tasks.
- Cardinal harder than parity:
  number of states influences hardness more than loops.
- Proportional quantifiers involve working-memory capacity.
- Humans are constrained by computational resources.

Perspectives

- Comprehension and brain?
- Comprehension strategies.
- Comprehension and working memory.
- Comprehension and monotonicity.
- Comprehension beyond quantifiers.

More details can be found in the paper on my web page:
http://staff.science.uva.nl/˜szymanik/

Thank you!