COMPREHENSION OF SIMPLE QUANTIFIERS
EMPIRICAL EVALUATION OF A COMPUTATIONAL MODEL

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Abstract

Comprehension of simple quantifiers in natural language.
Computational model posited by many logicians.
Linking computational complexity and cognitive science.
Comparing RT needed for understanding:
- FA-quantifiers vs. PDA-quantifiers;
- Aristotelian quantifiers vs. cardinal quantifiers;
- Parity quantifiers;
- PDA-quantifiers over ordered and unordered universes.
1. MOTIVATIONS

2. QUANTIFIERS AND AUTOMATA
   - Generalized Quantifiers
   - Automata for Quantifiers

3. THE EXPERIMENT
   - Comparing Quantifiers
   - Quantifiers and Ordering

4. CONCLUSIONS AND PERSPECTIVES
1 Motivations

2 Quantifiers and Automata
   - Generalized Quantifiers
   - Automata for Quantifiers

3 The Experiment
   - Comparing Quantifiers
   - Quantifiers and Ordering

4 Conclusions and Perspectives
A cognitive task is a computational task.
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Marr’s levels: computational, algorithmic, neurological.
Computability and Cognition

A cognitive task is a computational task.
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Today computational restrictions are taken seriously.
A cognitive task is a computational task.

Marr’s levels: computational, algorithmic, neurological.

Today computational restrictions are taken seriously.

Tsotsos, “Analyzing vision at the complexity level”, 1990

Frixione, “Tractable competence”, 2001

van Rooij, “The tractable cognition thesis”, 2008
A cognitive task is a computational task.

Marr’s levels: computational, algorithmic, neurological.

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- Tsotsos, “Analyzing vision at the complexity level”, 1990
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But not enough empirical links, too abstract considerations.
Meaning as Algorithm

- Ability of understanding sentences.
- Capacity of recognizing their truth-values.
Meaning as algorithm

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- Long-standing philosophical (Fregean) tradition.
- Meaning is a procedure for finding extension in a model.
Meaning as algorithm

- Ability of understanding sentences.
- Capacity of recognizing their truth-values.
- Long-standing philosophical (Fregean) tradition.
- Meaning is a procedure for finding extension in a model.
- Adopted often with psychological motivations.

- Suppes, “Variable-free semantics with remark on procedural extensions”, 1982
- Lambalgen & Hamm, “The proper treatment of events”, 2005
Brain activity during the comprehension of:

**FO-quantifiers vs. higher-order quantifiers.**
Previous investigations

Brain activity during the comprehension of:

FO-quantifiers vs. higher-order quantifiers.

Results:

- All quantities are associated with numerosity: recruit right inferior parietal cortex;
- Only higher-order activate working-memory capacity: recruit right dorsolateral prefrontal cortex;

McMillan et al., “Neural basis for generalized quantifiers comprehension”, 2005
Clark & Grossman, “Number sense and quantifier interpretation”, 2007
Corticobasal degeneration (CBD) — number knowledge.

Alzheimer (AD) and frontotemporal dementia (FTD) — working memory limitations.
Corticobasal degeneration (CBD) — number knowledge.
Alzheimer (AD) and frontotemporal dementia (FTD) — working memory limitations.
CBD impairs comprehension more than AD and FTD.
FTD and AD patients have greater difficulty in non-FO.

McMillan et al., “Quantifiers comprehension in corticobasal degeneration”, 2006
PROBLEMS

- Definability $\neq$ Complexity
- Computational differences missed;
- “Even” is higher-order but FA-computable.
- Complexity perspective is better grained.
- New experimental set up!


Szymanik and Zajenkowski, “Improving methodology of quantifier comprehension experiments”, 2009
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Motivations

Quantifiers and Automata

Generalized Quantifiers

Automata for Quantifiers

The Experiment

Comparing Quantifiers

Quantifiers and Ordering

Conclusions and Perspectives

Comprehension of simple quantifiers
**Simple Quantifier Sentences**

- Every poet has low self-esteem.
- Some dean danced nude on the table.
- At least 3 grad students prepared presentations.
- An even number of the students saw a ghost.
- Most of the students think they are smart.
- Less than half of the students received good marks.
LINDSTRÖM definition

**Definition**

A monadic generalized quantifier of type (1,1) is a class Q of structures of the form $M = (U, A_1, A_2)$, where $A_1, A_2 \subseteq U$. Additionally, Q is closed under isomorphism.
A FEW EXAMPLES

- some = \{ (U, A, B) : A, B \subseteq U \land A \cap B \neq \emptyset \}
A FEW EXAMPLES

- some = \{(U, A, B) : A, B \subseteq U \land A \cap B \neq \emptyset\}
- all = \{(U, A, B) : A, B \subseteq U \land A \subseteq B\}
A FEW EXAMPLES

- **some** = \{(U, A, B) : A, B \subseteq U \land A \cap B \neq \emptyset\}
- **all** = \{(U, A, B) : A, B \subseteq U \land A \subseteq B\}
- **exactly m** = \{(U, A, B) : A, B \subseteq U \land \text{card}(A \cap B) = m\}

- **even** = \{(U, A, B) : A, B \subseteq U \land \text{card}(A \cap B) = k \times 2\}
- **most** = \{(U, A, B) : \text{card}(A \cap B) > \text{card}(A \setminus B)\}
A FEW EXAMPLES

- **some** $= \{(U, A, B) : A, B \subseteq U \land A \cap B \neq \emptyset\}$
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A FEW EXAMPLES

- **some** = \{(U, A, B) : A, B ⊆ U ∧ A ∩ B ≠ ∅\}
- **all** = \{(U, A, B) : A, B ⊆ U ∧ A ⊆ B\}
- **exactly m** = \{(U, A, B) : A, B ⊆ U ∧ card(A ∩ B) = m\}
- **even** = \{(U, A, B) : A, B ⊆ U ∧ card(A ∩ B) = k × 2\}
- **most** = \{(U, A, B) : card(A ∩ B) > card(A − B)\}
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How do we encode models?

- Restriction to finite models of the form $M = (U, A, B)$. 

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Comprehension of simple quantifiers
**How do we encode models?**

- Restriction to finite models of the form $M = (U, A, B)$.
- List of all elements of the model: $c_1, \ldots, c_5$. 

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Comprehension of simple quantifiers
**Motivations**

**Quantifiers and Automata**

**The Experiment**

**Conclusions and Perspectives**

**Generalized Quantifiers**

**Automata for Quantifiers**

### How do we encode models?

- Restriction to finite models of the form $M = (U, A, B)$.
- List of all elements of the model: $c_1, \ldots, c_5$.
- Labeling every element with one of the letters: $a_{AB}, a_{A\bar{B}}, a_{\bar{A}B}, a_{AB}$, according to constituents it belongs to.

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Comprehension of simple quantifiers
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- Labeling every element with one of the letters: $a_{\bar{A} \bar{B}}, a_{\bar{A}B}, a_{AB}, a_{A\bar{B}}$, according to constituents it belongs to.
- Result: the word $\alpha_M = a_{\bar{A}B}a_{\bar{A}B}a_{A\bar{B}}a_{\bar{A}B}a_{\bar{A}B}$.

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Comprehension of simple quantifiers
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Generalized Quantifiers
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How do we encode models?

- Restriction to finite models of the form \( M = (U, A, B) \).
- List of all elements of the model: \( c_1, \ldots, c_5 \).
- Labeling every element with one of the letters: \( a_{\bar{A}B}, a_{\bar{A}B}, a_{\bar{A}B}, a_{AB}, \) according to constituents it belongs to.
- Result: the word \( \alpha_M = a_{\bar{A}B}a_{\bar{A}B}a_{AB}a_{\bar{A}B}a_{\bar{A}B} \).
- \( \alpha_M \) describes the model in which:
  - \( c_1 \in \bar{A}B \), \( c_2 \in A\bar{B} \), \( c_3 \in AB \), \( c_4 \in \bar{A}B \), \( c_5 \in \bar{A}B \).
How do we encode models?

- Restriction to finite models of the form $M = (U, A, B)$.
- List of all elements of the model: $c_1, \ldots, c_5$.
- Labeling every element with one of the letters: $a_{\overline{AB}}, a_{\overline{AB}}, a_{\overline{AB}}, a_{AB}$, according to constituents it belongs to.
- Result: the word $\alpha_M = a_{\overline{AB}}a_{\overline{AB}}a_{AB}a_{\overline{AB}}a_{\overline{AB}}$.
- $\alpha_M$ describes the model in which:
  $c_1 \in \overline{AB}, c_2 \in A\overline{B}c_3 \in AB, c_4 \in \overline{AB}, c_5 \in \overline{AB}$.
- The class $Q$ is represented by the set of words describing all elements of the class.
This model is uniquely described by $\alpha_M = a_{\overline{AB}}a_{\overline{AB}}a_{AB}a_{\overline{AB}}a_{\overline{AB}}$. 
ARISTOTELIAN QUANTIFIERS

"all", "some", "no", and "not all"

\[ L_{\text{All}} = \{ \alpha \in \Gamma^* : \#a_{\bar{A}\bar{B}}(\alpha) = 0 \} \]
CARDINAL QUANTIFIERS

E.g. “at least 3”, “at most 7”, and “between 8 and 11”

\[ \Gamma - \{ a_{AB} \} \quad \Gamma - \{ a_{AB} \} \quad \Gamma - \{ a_{AB} \} \quad \Gamma \]

\[ q_0 \xrightarrow{a_{AB}} q_1 \xrightarrow{a_{AB}} q_2 \xrightarrow{a_{AB}} q_3 \]

Finite automaton recognizing \( L_{\text{At least three}} \)

\[ L_{\text{At least three}} = \{ \alpha \in \Gamma^* : \#a_{AB}(\alpha) \geq 3 \} \]
Parity Quantifiers

E.g. “an even number”, “an odd number”

\[ L_{\text{Even}} = \{ \alpha \in \Gamma^* : \#a_{AB}(\alpha) \text{ is even} \} \]
E.g. “most”, “less than half”.

Most As are B iff \( \text{card}(A \cap B) > \text{card}(A - B) \).

\[ L_{\text{Most}} = \{ \alpha \in \Gamma^* : \#a_{AB}(\alpha) > \#a_{\overline{AB}}(\alpha) \} \].

There is no finite automaton recognizing this language.

We need internal memory.

A push-down automata will do.
**WHAT DOES IT MEAN THAT CLASS OF MONADIC QUANTIFIERS IS RECOGNIZED BY CLASS OF DEVICES?**

**Definition**

Let $\mathcal{D}$ be a class of recognizing devices, $\Omega$ a class of monadic quantifiers. We say that $\mathcal{D}$ accepts $\Omega$ if and only if for every monadic quantifier $Q$:

$$Q \in \Omega \iff \text{there is device } A \in \mathcal{D}(A \text{ accepts } L_Q).$$
**IN GENERAL**

<table>
<thead>
<tr>
<th>Definability</th>
<th>Examples</th>
<th>Recognized by</th>
</tr>
</thead>
<tbody>
<tr>
<td>FO</td>
<td>“all” “at least 3”</td>
<td>acyclic FA</td>
</tr>
<tr>
<td>FO($D_n$)</td>
<td>“an even number”</td>
<td>FA</td>
</tr>
<tr>
<td>PrA</td>
<td>“most”, “less than half”</td>
<td>PDA</td>
</tr>
</tbody>
</table>

Quantifiers, definability, and complexity of automata

- van Benthem, Essays in logical semantics, 1986
- Mostowski, Computational semantics for monadic quantifiers, 1998
OUTLINE

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4 CONCLUSIONS AND PERSPECTIVES
Joint work with Marcin Zajenkowski.
1st: RT in the comprehension of different quantifiers.
2nd: engagement of working-memory capacity.

- Szymanik and Zajenkowski, “Understanding quantifiers in language”, 2009
Motivations

Quantifiers and Automata

The Experiment

Conclusions and Perspectives

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Comprehension of simple quantifiers
GENERAL IDEA

Compare RT wrt the following classes of quantifiers:
GENERAL IDEA

- Compare RT wrt the following classes of quantifiers:
  - recognized by acyclic FA (first-order);
GENERAL IDEA

- Compare RT wrt the following classes of quantifiers:
  - recognized by acyclic FA (first-order);
  - not first-order recognized by FA (parity);
GENERAL IDEA

- Compare RT wrt the following classes of quantifiers:
  - recognized by acyclic FA (first-order);
  - not first-order recognized by FA (parity);
  - recognized by PDA but not FA.

Additionally:
- Aristotelian vs. cardinal quantifiers of higher rank.
- Troiani et al., "Is it logical to count on quantifiers? Dissociable neural networks underlying numerical and logical quantifiers", 2009
**GENERAL IDEA**

- Compare RT wrt the following classes of quantifiers:
  - recognized by acyclic FA (first-order);
  - not first-order recognized by FA (parity);
  - recognized by PDA but not FA.

- Additionally:
  - Aristotelian vs. cardinal quantifiers of higher rank.

Troiani et al., “Is it logical to count on quantifiers? Dissociable neural networks underlying numerical and logical quantifiers”, 2009
PREDICTIONS

- RT will increase along with the computational resources.
- Aristotelian qua. < parity qua. < proportional qua.
- Aristotelian qua. < cardinal qua. of high rank.
- Parity qua. < cardinal qua. of high rank.
PARTICIPANTS

- 40 native Polish-speaking adults (21 female).
- Volunteers: undergraduates from the University of Warsaw.
- The mean age: 21.42 years (SD = 3.22).
- Each participant tested individually.
Materials

80 grammatically simple propositions in Polish, like:

1. Some cars are red.
2. More than 7 cars blue.
3. An even number of cars is yellow.
4. Less than half of the cars are black.
More than half of the cars are yellow.

An example of a stimulus used in the first study
PROCEDURE

- 8 different quantifiers divided into four groups.
8 different quantifiers divided into four groups.

- “all” and “some”;

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**Comprehension of simple quantifiers**
8 different quantifiers divided into four groups.
- “all” and “some”;
- “odd” and “even”;
8 different quantifiers divided into four groups.

- “all” and “some”;
- “odd” and “even”;
- “less than 8” and “more than 7”;
- “less than half” and “more than half”.

Each quantifier was presented in 10 trials. The sentence true in the picture in half of the trials. Quantity of target items near the criterion of validation. Practice session followed by the experimental session. Each quantifier problem was given one 15.5 s event. Subjects were asked to decide the truth-value.
8 different quantifiers divided into four groups.

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Quantity of target items near the criterion of validation.

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- **Quantity of target items near the criterion of validation.**
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- Each quantifier was presented in 10 trials.
- The sentence true in the picture in half of the trials.
- **Quantity of target items near the criterion of validation.**
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- Subjects were asked to decide the truth-value.
ANALYSIS OF ACCURACY

<table>
<thead>
<tr>
<th>Quantifier group</th>
<th>Examples</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aristotelian FO</td>
<td>all, some</td>
<td>99</td>
</tr>
<tr>
<td>Parity</td>
<td>odd, even</td>
<td>91</td>
</tr>
<tr>
<td>Cardinal FO</td>
<td>less than 8, more than 7</td>
<td>92</td>
</tr>
<tr>
<td>Proportional</td>
<td>less than half, more than half</td>
<td>85</td>
</tr>
</tbody>
</table>

The percentage of correct answers
TO SUM UP

- Increase in RT was determined by the quantifier type
  \( F(2.4, 94.3) = 341.24; p < 0.001; \eta^2 = 0.90 \)

- Pairwise comparisons: all four types of quantifiers differed significantly from one another.

- The mean reaction time increased as follows: Aristotelian, parity, cardinal, proportional.
COMPARISON OF REACTION TIMES

Average reaction times in each type of quantifiers

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Comprehension of simple quantifiers
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GENERAL IDEA

- Investigating the role of working-memory capacity.
- The ordering as an additional independent variable.
- For example, consider the following sentence: “Most As are B.”
- Universe ordered in pairs \((a, b)\) such that \(a \in A, b \in B\).
PREDICTIONS

- Given “good” ordering WM capacity is not needed.
- Ordering simplifies the problem = decrease in RT.
PARTICIPANTS

- 30 native Polish-speaking adults (18 females).
- Undergraduates from two Warsaw universities.
- The mean age: 23.4 years (SD = 2.51).
- Each subject tested individually.
Materials and procedure

- 16 grammatically simple propositions in Polish.
- E.g. “More than half of the cars are blue”.
- A car park with 11 cars.
- 2 quantifiers: “less than half” and “more than half”.
- Presented to each subject in 8 trials.
- Each type of sentence true in half of the trials.
- 4 ordered and 4 unordered pictures.
- The rest of the procedure the same as before.
EXAMPLE OF AN ORDERED TASK

More than half of the cars are red.

A case when cars are ordered

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Comprehension of simple quantifiers
**EXAMPLE OF AN UNORDERED TASK**

*More than half of the cars are green.*

A case when cars are distributed randomly.
RESULTS

- Higher accuracy of judgments for ordered universes (89%);
- Than for unordered (79%).
- Proportional quantifiers over randomized universes (M=6185.93; SD=1759.09);
- Over ordered models (M=4239.00; SD=1578.26);
- Hypothesis confirmed! \( t(29) = 5.87; p < 0.001; d = 1.16 \).
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CONCLUSIONS

- Plausibility of the model.
- Aristotelian easier than parity: loops influence the complexity of cognitive tasks.
- Cardinal harder than parity: number of states influences hardness more than loops.
- Proportional quantifiers involve working-memory capacity.
- Humans are constrained by computational resources.
Perspectives

- Comprehension and brain?
Comprehension and brain?
Comprehension strategies?
Comprehension and brain?
Comprehension strategies?
Comprehension and working memory?
Perspectives

- Comprehension and brain?
- Comprehension strategies?
- Comprehension and working memory?
- Comprehension and monotonicity?
Perspectives

- Comprehension and brain?
- Comprehension strategies?
- Comprehension and working memory?
- Comprehension and monotonicity?
- Comprehension beyond quantifiers?
Thank you!