A Remark on Collective Quantification

Juha Kontinen[1] Jakub Szymanik[2]

[1]Department of Mathematics and Statistics
University of Helsinki

[2]Institute for Logic, Language and Computation
Universiteit van Amsterdam

The 9th Szklarska Poręba Workshop
February 22, 2008

ABSTRACT

The common strategy in formalizing collective quantification has been to define the meanings of collective determiners using certain type-shifting operations. These type-shifting operations, i.e., lifts, define the collective interpretations of determiners systematically from the standard meanings of quantifiers. We argue that this approach is probably not expressive enough to formalize all collective quantification in natural language!

Outline

MOTIVATIONS

- Complexity depends on the quantifiers.
- Mainly distributive determiners are considered.
- However, plural objects are becoming important.
- E.g. in game-theory, where groups of agents are acting.

TWO EXAMPLES

(1.) Tikitu and Samson lifted the poker table together.

(2.) Most groups of students played Hold'em together.

OUTLINE

Let's start with examples

(1.) Five people lifted the table.

(1'.) $\exists^{=5} x[\text{People}(x) \wedge \text{Lift}(x)]$.

(1''.) $\exists X[X \subseteq \text{People} \wedge \text{Card}(X) = 5 \wedge \text{Lift}(X)]$.

(2.) Some students played poker together.

(2'.) $\exists X[X \subseteq \text{Students} \wedge \text{Play}(X)]$.

Existential modifier

Definition

Fix a universe of discourse U and take any $X \subseteq U$ and $Y \subseteq \mathcal{P}(U)$. Define the existential lift Q^{EM} of a quantifier Q in the following way:

$$\mathrm{Q}^{EM}(X, Y) \text{ is true } \iff \exists Z \subseteq X[\mathrm{Q}(X, Z) \wedge Z \in Y].$$

$$((et)((et)t)) \rightsquigarrow ((et)(((et)t)t))$$

Determiner fitting

Definition

For all $X, Y \subseteq \mathcal{P}(U)$ we have that

$$\mathrm{Q}^{\mathrm{dfit}}(X, Y) \text{ is true}$$

$$\iff$$

$$\mathrm{Q}[\cup X, \cup(X \cap Y)] \wedge [X \cap Y = \emptyset \vee \exists W \in X \cap Y \wedge \mathrm{Q}(\cup X, W)].$$

$$((et)((et)t)) \rightsquigarrow (((et)t)(((et)t)t))$$

OUTLINE

LINDSTRÖM QUANTIFIERS

$$
\begin{array}{rcl}
\forall & = & \{(M, P) \mid P = M\}. \\
\exists & = & \{(M, P) \mid P \subseteq M \;\&\; P \neq \emptyset\}. \\
\text{even} & = & \{(M, P) \mid P \subseteq M \;\&\; \text{card}(P) \text{ is even}\}. \\
\text{most} & = & \{(M, P, S) \mid P, S \subseteq M \;\&\; \text{card}(P \cap S) > \text{card}(P - S)\}. \\
\text{some} & = & \{(M, P, S) \mid P, S \subseteq M \;\&\; P \cap S \neq \emptyset\}.
\end{array}
$$

SECOND-ORDER GQS

$$
\begin{aligned}
\exists^2 &= \{(M,P) \mid P \subseteq \mathcal{P}(M) \ \& \ P \neq \emptyset\}. \\
\textsc{Even} &= \{(M,P) \mid P \subseteq \mathcal{P}(M) \ \& \ \mathsf{card}(P) \text{ is even}\}. \\
\textsc{Even}' &= \{(M,P) \mid P \subseteq \mathcal{P}(M) \ \& \ \forall X \in P(\mathsf{card}(X) \text{ is even})\}. \\
\textsc{Most} &= \{(M,P,S) \mid P,S \subseteq \mathcal{P}(M) \ \& \ \mathsf{card}(P \cap S) > \mathsf{card}(P - S)\}.
\end{aligned}
$$

WARNING

Do not confuse:

- FO GQs (Lindström) with FO-definable quantifiers

  E.G. most is FO GQs but is not FO-definable.
- SO GQs with SO-definable quantifiers

  E.G. MOST is SO GQs but probably not SO-definable.

OUTLINE

For example . . .

Definition

Denote by some^{EM}:

$$\{(M, P, G) \mid P \subseteq M;\ G \subseteq \mathcal{P}(M) :\ \exists Y \subseteq P(Y \neq \emptyset \ \&\ P \in G)\}.$$

(3.) Some students played poker together.

(3'.) some^{EM} x, X[Student(x), Play(X)].

ANOTHER EXAMPLE ...

DEFINITION

We take five^{EM} to be the second-order quantifier denoting:

$$\{(M, P, G) \mid P \subseteq M;\ G \subseteq \mathcal{P}(M) :\ \exists Y \subseteq P(\mathrm{card}(Y) = 5\ \&\ P \in G)\}.$$

(4.) Five people lifted the table.

(4'.) five$^{EM}x, X[\mathrm{Student}(x), \mathrm{Lift}(X)]$.

General observation

Theorem

Let Q be a first-order quantifier definable in SO. Then the second-order quantifiers Q^{EM} and Q^{dfit} are definable in SO.

Proof.

Let us consider the case of Q^{EM}. Let $\psi(x)$ and $\phi(Y)$ be formulas. We want to express $Q^{EM}x, Y(\psi(x), \phi(Y))$ in second-order logic. By the assumption, the quantifier Q can be defined by some sentence $\theta \in SO[\{P_1, P_2\}]$. We can now use the following formula:

$$\exists Z(\forall x(Z(x) \to \psi(x)) \wedge (\theta(P_1/\psi(x), P_2/Z) \wedge \phi(Y/Z)).$$

$\square$

Outline

Collective Most

(5.) Most groups of students played Hold'em together.

(5'.) $\textsc{Most}\, X, Y\, [\text{Students}(X), \text{Play}(Y)]$.

- The discussed lifts do not give the intended meaning.
- It is unlikely that *any* lift can do the job.
- Otherwise, highly unexpected things would happen!

The main theorem

Theorem

If the quantifier Most *is definable in second-order logic, then counting hierarchy,* CH *is equal polynomial hierarchy,* PH*. Moreover,* CH *collapses to its second level.*

Proof.

The logic FO(Most) can define complete problems for each level of the CH (Kontinen&Niemisto'06). If Most would be definable in SO, then FO(Most) $\leq$ SO and therefore SO would contain complete problems for each level of the CH. This would imply that CH $=$ PH and furthermore that CH $\subseteq$ PH $\subseteq C_2P$. □

OUTLINE

To sum up

- We can approach collectivity in terms of SO GQs.
- The previous attempts have relied on SO-definable GQs .
- This approach is probably not general enough.
- We observed this by studying computational complexity.

MORE DETAILS IN:

J. Kontinen and J. Szymanik
A Remark on Collective Quantification,
Journal of Logic, Language and Information , Volume 17, Number 2, 2008, pp. 131–140.