

OUTLINE OF THE PROJECT

QUANTIFIERS, GAMES, AND COMPLEXITY

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OUTLINE

- 1 GENERALIZED QUANTIFIERS
- 2 BRANCHING QUANTIFIERS
- 3 COMPLEXITY AND DIFFICULTY

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INSTEAD OF INTRODUCTION

- Every poet has low self-esteem.
- Some dean danced nude on the table.
- At least 3 grad students prepared presentations.
- An even number of the students saw a ghost.
- Most of the students think they are smart.
- Less than half of the students received good marks.
- An equal number of logicians, philosophers, and linguists climbed Elbrus.



LINDSTRÖM DEFINITION

DEFINITION

A generalized quantifier is a class Q of structures of a finite relational signature which is closed under isomorphism. The type of Q can be identified with a finite sequence (n_1, \dots, n_k) of natural numbers.



FEW EXAMPLES TO MAKE IT CLEAR

- $K_{\exists} = \{(|M|, R) : R \subseteq |M| \wedge R \neq \emptyset\}$.
- $K_{\forall} = \{(|M|, R) : R = |M| \wedge R \neq \emptyset\}$.
- $K_{\exists=m} = \{(|M|, R) : R \subseteq |M| \wedge \text{card}(R) = m\}$.
- $K_{D_n} = \{(|M|, R) : R \subseteq |M| \wedge \text{card}(R) = kn\}$.
- $K_{Most} = \{(|M|, R_1, R_2) : \text{card}(R_1 \cap R_2) > \text{card}(R_1 - R_2)\}$.
- $K_{Equal} = \{(|M|, R_1, \dots, R_n) : \text{card}(R_1) = \dots = \text{card}(R_n)\}$.



GAMES FOR ELEMENTARY QUANTIFIERS

- If $\psi := \exists x\varphi(x)$, then Eloise chooses an element $d \in |M|$ and the game continues for the formula $\varphi(d)$.
- If $\psi := \forall x\varphi(x)$, then Abelard chooses an element $d \in |M|$ and the game continues for the formula $\varphi(d)$.
- If $\psi := \exists^m x\varphi(x)$, then Eloise chooses subset $A \subseteq M$, such that $\text{card}(A) = m$, and Abelard chooses $d \in A$ and the game continues for the formula $\varphi(d)$.



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HINTIKKA'S-LIKE SENTENCES

- 1 Some relative of each villagers and some relative of each townsmen hate each other.
- 2 Most villagers and most townsmen hate each other.
- 3 Exactly half of all villagers and exactly half of all townsmen hate each other.



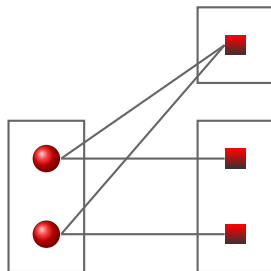
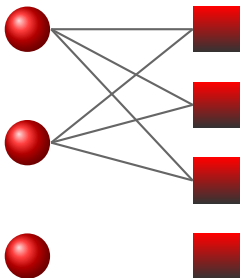
HINTIKKAS' S THESIS

Hintikka claims that **we need branching quantifiers to express their meaning.**

- 1 $\forall x \exists y \forall z \exists w ((V(x) \wedge T(z)) \Rightarrow (R(x, y) \wedge R(z, w) \wedge H(y, w))).$
- 2 $\exists f \exists g \forall x \forall z ((V(x) \wedge T(z)) \Rightarrow (R(x, f(x)) \wedge R(z, g(z)) \wedge H(f(x), g(z))))).$
- 3 $\text{MOST } x : V(x) \text{ MOST } y : T(y) H(x, y).$
- 4 $\exists A \exists B [\text{MOST } x (V(x), A(x)) \wedge \text{MOST } y (T(y), B(y)) \wedge \forall x \forall y (A(x) \wedge B(y) \Rightarrow H(x, y))].$



ILLUSTRATIONS



GTS AND SUBGAME SEMANTICS I

- If $\psi := \forall x \exists y \forall z \exists w \varphi(x)$ then Abelard chooses an element $a \in |M|$ and Eloise chooses an element $b \in |M|$, and then Abelard chooses $c \in |M|$ and Eloise chooses **independently** $d \in |M|$.
- GTS is counterintuitive, for instance $\varphi \vee \varphi$, $\varphi \wedge \varphi$, and φ are not equivalent.

OBJECTIVE

Investigate subgame semantics as an alternative. Compare it with strategic interpretation of Henkin quantifiers.



GTS AND SUBGAME SEMANTICS II

OBJECTIVE

Formulate game-theoretical (subgame) semantics for all branching quantifiers.

OBJECTIVE

Investigate linguistic plausibility of various interpretations for branching sentences in natural language.



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MONADIC QUANTIFIERS AND AUTOMATA

definability	example	recognized by
FO	exactly 6	acyclic FA
$FO(D_n)$	even	FA
Pr	most	PDA

TABLE: Quantifiers and complexity of corresponding algorithms.

Important: FA do not have a memory, PDA have stack - which is considered a form of memory.



NEUROIMAGING STUDY

- Comprehension of FO and non-FO quantifiers recruit right inferior parietal cortex – the region of brain associated with number knowledge.
- Non-FO quantifiers recruit right dorsolateral prefrontal cortex – the part of brain associated with executive resources and working memory.

OBJECTIVE

Find psychologically plausible explanation of these results.



COMPLEXITY OF BRANCHING QUANTIFIERS

THEOREM

Henkin quantifier defines NP-complete class of finite models.

THEOREM

Branching MOST defines NP-complete class of finite models.

OBJECTIVE

What is the source of such complexity of those constructions?

THEOREM

Ramsey quantifiers define NP-complete class of finite models.



COMPLEXITY, DIFFICULTY AND GAMES

OBJECTIVE




Study evaluation games in connection with the way people understand quantifier sentences.

OBJECTIVE

Try to use higher-order games, like signaling games, to investigate connection between difficulty and complexity.



FOR FURTHER READING I

-  **T. Janssen**
Independent choices and the interpretation of IF-logic.
JOLLI 11: 2002.
-  **M. Mostowski, J. Szymanik**
Semantical bounds for everyday language.
Semiotica, to appear.
-  **N. Gierasimczuk, J. Szymanik**
Hintikka's Thesis Revisited.
preliminary report, see: ILLC Preprint Series, 2006.



FOR FURTHER READING II



C. McMillan et al.

Neural Basis for Generalized Quantifiers.

Neuropsychologia, 43,2005.



M. Sevenster

Branches of imperfect information: logic, games, and computation.

PhD Thesis, ILLC 2006.



J. Szymanik

A note on some neuroimaging study of natural language quantifiers comprehension.

Neuropsychologia, to appear.

