COMPLEXITY OF QUANTIFIERS MEANING

HOW TO APPLY DESCRIPTIVE COMPLEXITY THEORY TO NATURAL LANGUAGE SEMANTICS?

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1st June 2007

GOAL OF THE TALK

To convince you — by giving few nontechnical examples — that descriptive complexity theory may be fruitfully used in natural language semantics.

Outline

Meaning as an algorithm

- Meaning of φ is “the mode of presenting” its truth-value.
- Meaning $\equiv$ algorithm.
- Classifying expressions by complexity of their meanings.

Descriptive complexity

Computational complexity and expressibility.

Input M and φ

Problem $M \models \varphi$?

Complexity w.r.t. the size of the model.

Theorem

FO $\subseteq$ *PTIME*

Theorem (Fagin, 1974)

Σ^1_1 *captures NPTIME.*

Outline

Monadic quantifiers

Definition

Monadic generalized quantifiers are relations between subsets of the universes which are closed under permutations.

- Every poet has low self-esteem.
- At least 3 grad students prepared presentations.
- An even number of the students saw a ghost.
- Most of the students think they are smart.

MONADIC QUANTIFIERS AND AUTOMATA

definability	example	recognized by
FO	exactly 6	acyclic FA
$FO(D_n)$	even	FA
Pr	most	PDA

TABLE: Quantifiers and complexity of corresponding algorithms.

FA do not have a memory.
PDA have stack - which is considered a form of memory.

NEUROIMAGING EVIDENCE

- Comprehension of all quantifiers recruit right inferior parietal cortex – the region of brain associated with number knowledge.
- Non-FO quantifiers recruit right dorsolateral prefrontal cortex – the part of brain associated with executive resources and working memory.

Outline

Branching quantifiers

1. Some relative of each villagers and some relative of each townsmen hate each other.
2. Most villagers and most townsmen hate each other.
3. Exactly half of all villagers and exactly half of all townsmen hate each other.

Hintikkas's Thesis Revisited

1. $\begin{matrix}\forall x \exists y \\ \forall z \exists w\end{matrix} ((V(x) \wedge T(z)) \Rightarrow (R(x,y) \wedge R(z,w) \wedge H(y,w)))$.
2. $\begin{matrix}\text{MOST } x : V(x) \\ \text{MOST } y : T(y)\end{matrix} H(x,y)$.

Theorem

Both these formulae are NP-complete.

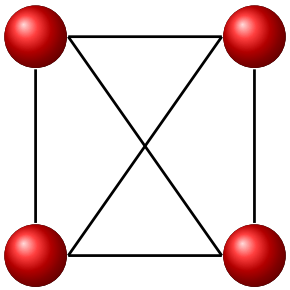
Claim

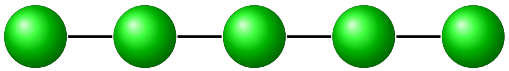
It is too difficult for direct verification!

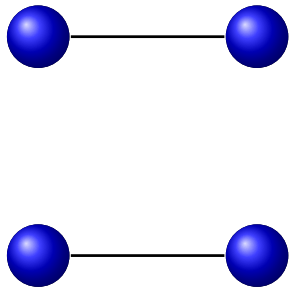
Outline

STRONG, INTERMEDIATE AND WEAK RECIPROCITY

1. Most members of the parliament refer to each other indirectly.
2. Most Boston pitchers sat alongside each other.
3. Most Pirates of the Carribean were staring at each other.







Strong Meaning Hypothesis

Hypothesis

Reading associated with the reciprocal in a given sentence is the strongest available reading which is consistent with relevant information supplied by the context.

Is there any computational explanation for this?

SMH EXPLAINED

THEOREM

Let φ be any reciprocal sentence with MOST*.*
If φ is interpreted strong, then it is NP-complete.
Otherwise, it is in P.

- Dalrymple et al. distinction is robust.
- Sometimes strong interpretation is cognitively too hard.
- Subjects shift to easier interpretations.
- Easier interpretations are implied.
- Computational perspective is consistent with SMH.

Outline

What are NP computations?

- Choose certificate (proof).
- Check in PTIME its correctness.
- Accept if it is correct.

INDIRECT VERIFICATION

(1.) *There were more boys than girls at the party.*

(2.) *At the party every girl was paired with a boy.*

(3.) *Peter came alone to the party.*

(4.) *Most villagers are A.*

(5.) *Most townsmen are B.*

(6.) *All A and all B hate each other.*

(7.) *Most villagers and most townsmen hate each other.*

General hypothesis

Hypothesis

Everyday language is bounded by the Σ^1_1-properties.

One consequence:

- Σ^1_1 is not closed on negation.
- Sentence belongs to everyday language – its negation not.

Outline

Descriptive complexity in linguistics

- Measure complexity of language constructions.
- Already there are some links with cognitive psychology.
- It is one of the factors in looking for possible meanings.
- It may be used to explain pragmatic phenomena.
- It is interesting aspect of language theory on its own.
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