# Monotonicity in quantifier verification

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# Outline

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# NL determiners

1. All poets have low self-esteem.
2. Some dean danced nude on the table.
3. At least 3 grad students prepared presentations.
4. An even number of the students saw a ghost.
5. Most of the students think they are smart.
6. Less than half of the students received good marks.

# Monotone quantifiers

## Definition

Q is upward monotone if $X \subseteq Y$, then Q($X$) entails Q($Y$).

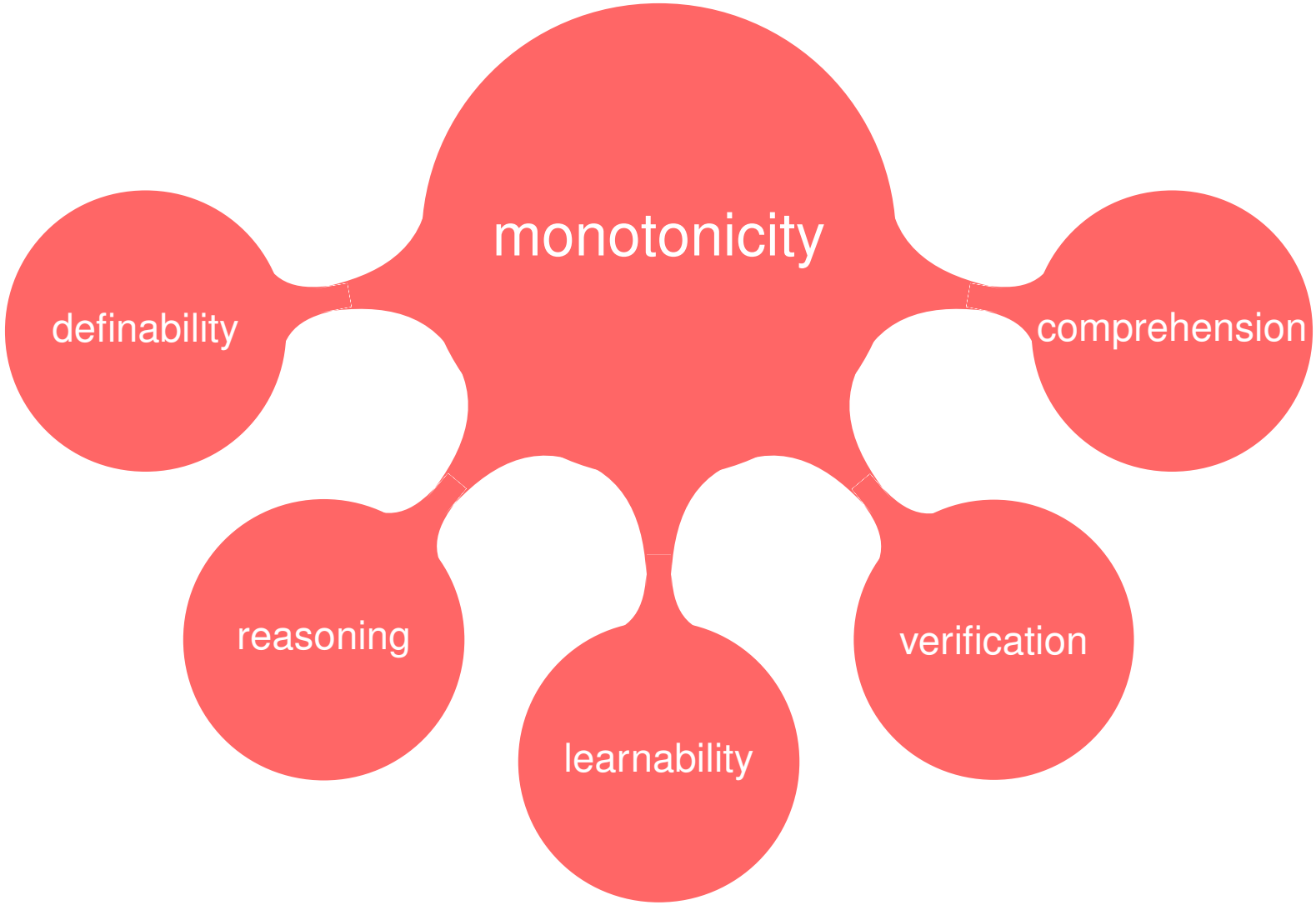
1. Every boy runs fast.
2. Every boy runs.

## Definition

Q is downward monotone if $Y \subseteq X$, then Q($X$) entails Q($Y$).

1. No boy runs.
2. No boy runs fast.

# Monotonicity – key property in logic and language



# Outline

# Just and Carpenter 1971

## Observation

*Processing time of negative quantifiers is greater than processing time of affirmative quantifiers.*

Just & Carpenter, Comprehension of negation with quantification, Journal of Verbal Learning and Verbal Behavior, 1971

# 3 kinds of sentences

1. Syntactic negatives with particle:
   - The dots are red.
   - The dots aren't red.
2. Syntactic negatives without particle:
   - Many of the dots are red.
   - Few of the dots are red.
3. Semantic negatives:
   - A majority of the dots are red.
   - A minority of the dots are red.

# Only some pairs contrasted w.r.t. monotonicity:

1. All of the dots are red.
2. None of the dots are red.

Most of the material was based on negativity vs. affirmativity.

# Affirmativity and monotonicity?

- Monotonicity is a semantic property of quantifiers;
- Degree of affirmativity is a linguistic concept, e.g.,:
  - tag test;
  - licensing NPIs.

## Example

1. Few children are dirty, are they?
2. Few children believe that any more.
3. *A few children are dirty, are they?
4. *A few children believe that any more.

# A partial dissociation

## Observation

*Dissociation between downward monotonicity and negativity.*

## Example

1. At most half of the children believe that, don't they?
2. Not many children believe that, do they?

## Observation

*Downward monotone quantifiers fall into two classes: affirmatives and negatives.*

# The confound

## Question

*Are effects reported by JC'71 due to monotonicity or negativity?*

- It's common to mix up monotonicity and negativity.
- Our aim is to clearly separate those two:
  - Polish has no negativity degrees.
- Focus on monotonicity.

# Outline

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# Comparison model

4 stage processing of the comparison model:

1. Sentence encoding,
2. Picture encoding,
3. Comparing,
4. Responding.

Component latencies are additive.

Clark & Chase, On the process of comparing sentences against pictures, Cognitive Psychology, 1972

# Pros & Cons of the comparison model

## Pros

- *Explains variations w.r.t. negativity:*
  - *negatives harder than affirmatives;*
  - *true affirmatives easier than false ones;*
  - *false negatives easier than true ones.*

## Cons

- *How to cover quantifiers?*
- *It says very little about monotonicity.*
- *Arbitrary psychological representation.*
- *Little insight into the actual computational process.*

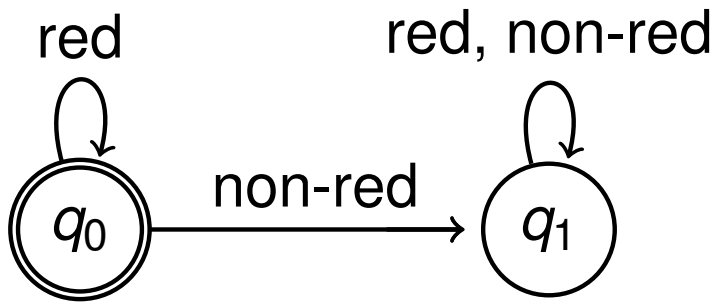
# Outline

# Simplicity

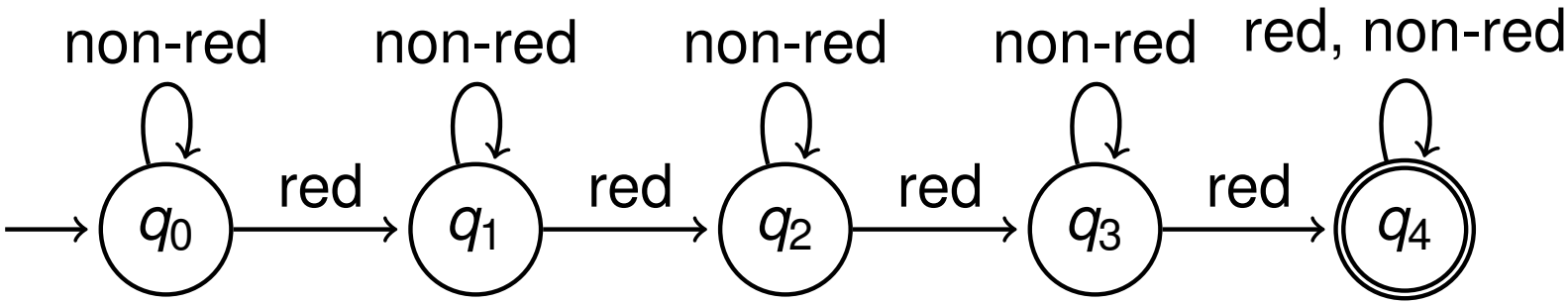
- Simple quantifiers can be computed by simple automata.
- Encoding natural counting strategies.
- We restrict ourselves to precise counting.

van Benthem, Essays in logical semantics, 1986

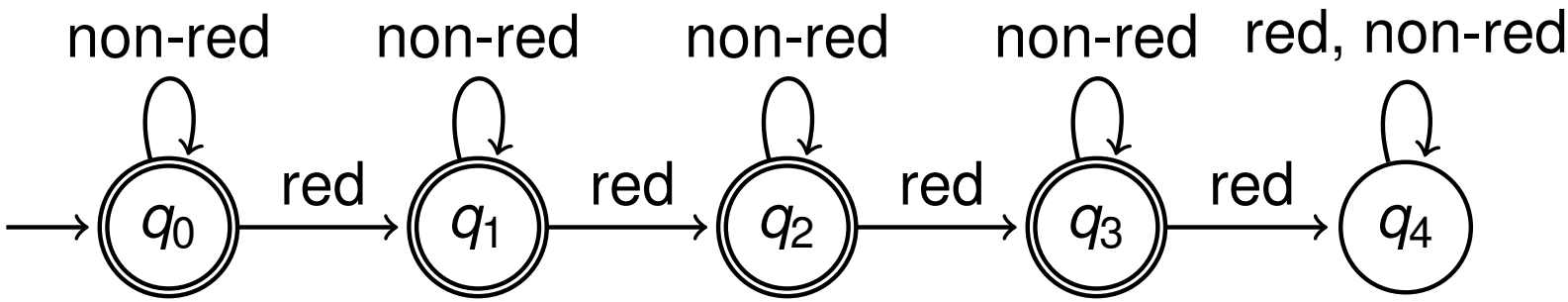
# Every dot is red.



# More than 3 dots are red.

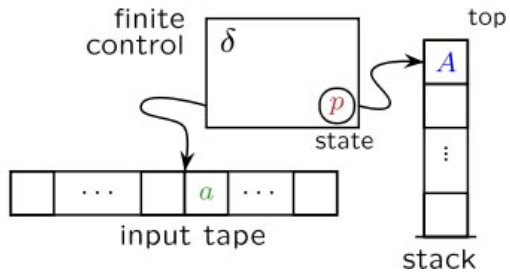


# Fewer than 4 dots are red.



# Proportional quantifiers

1. More than half of the dots are red.
2. Fewer than half of the dots are red.




- Not computable by finite-automata.
- We need working memory.
- Simple push-down automata will do.

# Psychological plausibility

## Observation

*The more complex automata the longer reaction time and greater working memory involvement.*



McMillan et al., Neural basis for generalized quantifiers comprehension, Neuropsychologia, 2005


Szymanik, A Note on some neuroimaging study of natural language quantifiers comprehension, Neuropsychologia, 2007


Szymaniki & Zajenkowski, Comprehension of simple quantifiers. Empirical evaluation of a computational model, Cognitive Science, 2010


Szymanik & Zajenkowski, Quantifiers and working memory, LNCS, 2010

# Outline

# Design

- 4 different quantifiers:
  - Cardinal quantifiers (“more than 7”,“fewer than 8”);
  - Proportional (“more than half”, “fewer than half”).
- Upward monotone vs. downward monotone.
- True vs. false.
- Subjects were timed when asked to decide if true.

# Predictions

1. RT increase along with the computational complexity.
2. Complexity influenced by (monotonicity $\times$ truth-value):
   - In the case of the cardinal sentences,
   - but not the proportional sentences.

# Predictions in details

1. “More than 7”: true > false (8>7).
2. “Fewer than 8”: true < false (7<8).
3. On average “fewer than 8” > “more than 7”.
4. No difference between proportional quantifiers.
5. Proportional quantifiers > cardinal quantifiers.

# Outline

# 2 experiments

- 2 independent experiments.
- Same results.
- We report only on the first one.

# Participants

- 30 native Polish-speaking adults (15 female).
- Volunteers: undergraduates from the University of Warsaw.
- The mean age: 21.30 years (SD = 2.54).
- Each participant tested individually.

# Materials

16 grammatically simple propositions in Polish, like:

1. More than 7 cars are blue.
2. Fewer than 8 cars are yellow.
3. More than half of the cars are red.
4. Fewer than half of the cars are black.

# Materials continued

*More than half of the cars are yellow.*

An example of a stimulus used in the first study

# Procedure

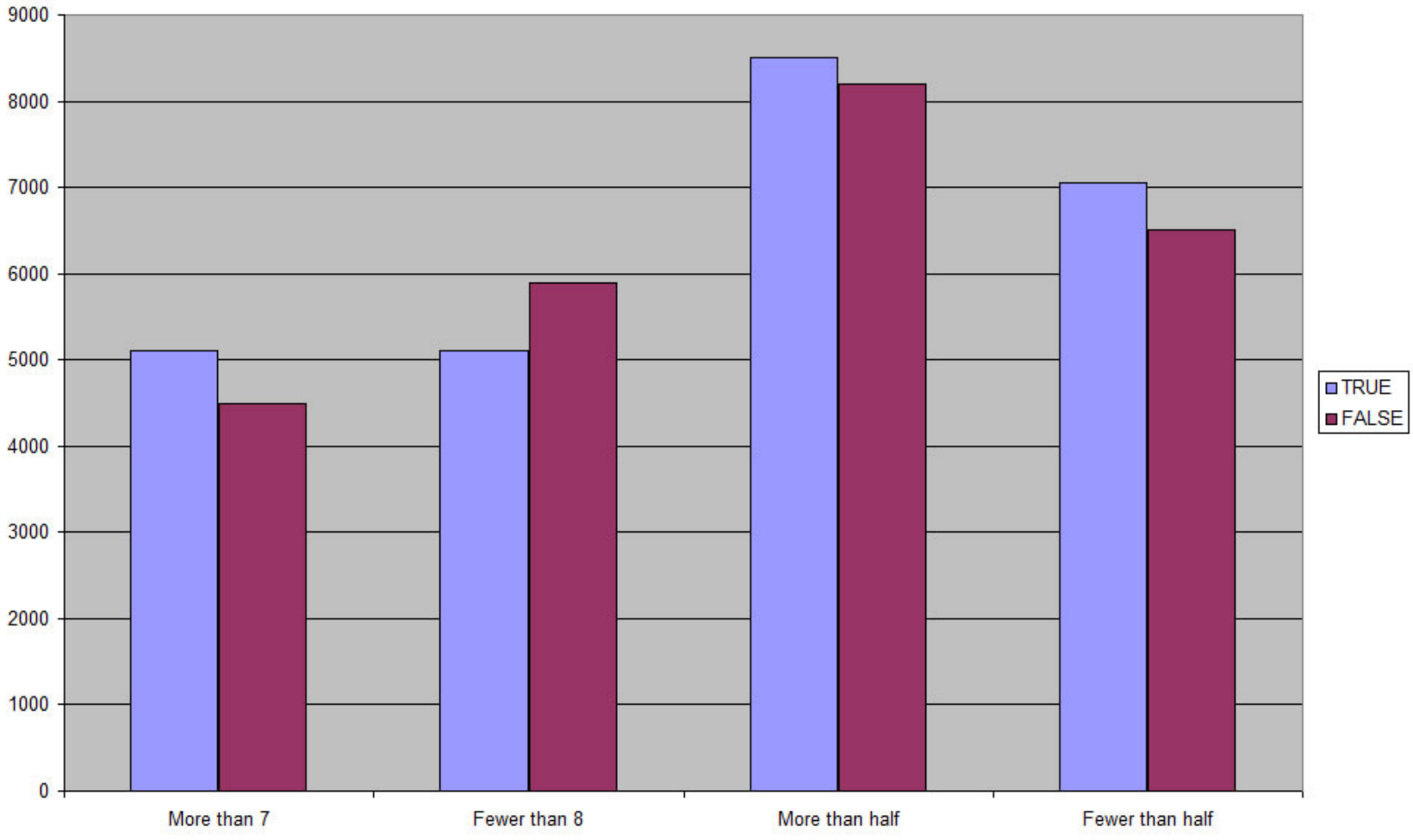
- Each quantifier was presented in 10 trials.
- The sentence true in the picture in half of the trials.
- Quantity of target items near the criterion of validation.
- Practice session followed by the experimental session.
- Each quantifier problem was given one 15.5 s event.
- Subjects were asked to decide the truth-value.

# Outline

# Raw data

| Quantifier | | M | SD |
|---|---|---|---|
| More than 7 | True | 5073.36 | 1244 |
| | False | 4526.36 | 1299 |
| | Overall | 4799.87 | 1157 |
| Fewer than 8 | True | 5047.80 | 1693 |
| | False | 5872.13 | 1795 |
| | Overall | 5459.97 | 1510 |
| More than half | True | 7431.12 | 2076 |
| | False | 7195.13 | 2342 |
| | Overall | 7313.42 | 2016 |
| Fewer than half | True | 7064.80 | 2433 |
| | False | 6539.67 | 2092 |
| | Overall | 6802.23 | 2098 |

# Interaction effect



# Outline

# cf. Just and Carpenter, 1971

- Difference between cardinal quantifiers;
- No difference between proportional quantifiers.
- Differences in JC'71 is due to negativity.
- Vague quantifiers of JC'71 trigger different strategies.

# False/True interaction

- $RT(\exists_T^{\geq 7}) > RT(\exists_F^{\geq 7})$? 👍
- $RT(\exists_T^{\leq 8}) < RT(\exists_F^{\leq 8})$. 👍
- No difference in proportional quantifiers. 👍
- Complexity hypothesis. 👍

# Model's plausibility

- Complexity hypothesis derived from the minimal automata.
- Proportional quantifiers > numerical quantifiers.
- Monotonicity accounts for 45% of the variance.
- Complexity for over 90%.
- Complexity explains more than monotonicity.

# Thank you for attention