Lecture 2: Quantifiers and Approximation
Case study: Most vs More than half

Jakub Szymanik
Outline

Number Sense
  Approximate Number Sense

Approximating ‘most’

Superlative Meaning of ‘most’

What About Counting?
Outline

Number Sense
Approximate Number Sense

Approximating ‘most’

Superlative Meaning of ‘most’

What About Counting?
Observation

Quantifiers are associated with truth-conditions.
Observation
Quantifiers are associated with truth-conditions.

Observation
Quantifiers are associated with verification strategies.
Number knowledge

Observation

*Quantifiers embed number knowledge in language.*
Number knowledge

Observation
Quantifiers embed number knowledge in language.

Question
What is the characteristics of that ability in humans?
Number Sense

Definition
An intuitive understanding of numbers, their magnitude, relationships, and how they are affected by arithmetical operations.

Dehaene, The Number Sense: How the Mind Creates Mathematics, 1999
Two Core Systems of Numbers

1. Approximate representations
2. Precise representations

Feigenson et al., Core Systems of Numbers, Trends in Cognitive Science, 2004
Outline

Number Sense
Approximate Number Sense

Approximating ‘most’

Superlative Meaning of ‘most’

What About Counting?
Test Your Approximate Number Sense

Test
Approximate Number Sense (ANS)
Approximate Number Sense (ANS)
ANS Cont.

- common to many nonverbal animals;
ANS Cont.

- common to many nonverbal animals;
- an evolutionary ancient cognitive resource;
ANS Cont.

- common to many nonverbal animals;
- an evolutionary ancient cognitive resource;
- generates representations of numerosity;
ANS Cont.

- common to many nonverbal animals;
- an evolutionary ancient cognitive resource;
- generates representations of numerosity;
- across multiple modalities;
common to many nonverbal animals;
an evolutionary ancient cognitive resource;
generates representations of numerosity;
across multiple modalities;
doesn’t require explicit training;
ANS Cont.

► common to many nonverbal animals;
► an evolutionary ancient cognitive resource;
► generates representations of numerosity;
► across multiple modalities;
► doesn’t require explicit training;
► but works within certain limits.
Distance-effect
Distance-effect
Distance-effect
Distance-effect
Observation
There is a systematic, monotonous decrease in numerosity discrimination performance as the numerical distance between the numbers decreases.

Example
From 6 vs 18 to 6 vs 12.
Size-effect
Size-effect
Size-effect
Size-effect
Observation

For equal numerical distance, performance also decreases with increasing number size.
Observation
For equal numerical distance, performance also decreases with increasing number size.

Example
5 things are detectable different from 10 then 15 from 20.
Weber’s Law

*Discriminability depends on the ratio of relevant representational values.*
Weber’s Law

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\[
WR = \frac{\text{larger set}}{\text{smaller set}}
\]
Weber’s Law

Discriminability depends on the ratio of relevant representational values.

\[ \text{WR} = \frac{\text{larger set}}{\text{smaller set}} \]

It explains ratio dependent performance.
Mental Number Line

The more overlap, the poorer the discriminability
Development of ANS

- The acuity of ANS improves during childhood.
Development of ANS

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- Adults can discriminate 7:8 ratios.
Development of ANS

- The acuity of ANS improves during childhood.
- Adults can discriminate 7:8 ratios.
- But 6-months infants only 1:2.
ANS and Counting

- By 5yrs ANS is mapped onto discrete number words.
ANS and Counting

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- It is activated anytime we deal with numbers.
ANS and Counting

- By 5yrs ANS is mapped onto discrete number words.
- It is activated anytime we deal with numbers.
- Distance and size effect for Arabic numerals!
Outline

Number Sense
  Approximate Number Sense

Approximating ‘most’

Superlative Meaning of ‘most’

What About Counting?
Meanings of ‘most’

most\[A, B]\] = 1 iff \(\text{card}(A \cap B) > \frac{\text{card}(A)}{2}\)
Meanings of ‘most’

$$\text{most}[A, B] = 1 \text{ iff } \text{card}(A \cap B) > \frac{\text{card}(A)}{2}$$

$$\text{most}[A, B] = 1 \text{ iff } \text{card}(A \cap B) > \text{card}(A - B)$$
Yet Another Meaning of ‘most’

$$\text{most}[A, B] = 1 \text{ iff } \text{OneToOnePlus}(A \cap B, A - B)$$

Definition

$$\text{OneToOnePlus}(X, Y) \iff \exists X' \subseteq X \text{ s.t. there is a one-to-one function between } X' \text{ and } Y \text{ but not } X \text{ and } Y.$$
Corresponding Procedures

They suggest different strategies:
Corresponding Procedures

They suggest different strategies:

▶ Most \([A, B] = 1\) iff \(\text{card}(A \cap B) > \frac{\text{card}(A)}{2}\)

▶ Comparing number of target dots to the half of all dots.
Corresponding Procedures

They suggest different strategies:

- $\text{most}[A, B] = 1$ iff $\text{card}(A \cap B) > \frac{\text{card}(A)}{2}$
- Comparing number of target dots to the half of all dots.
- $\text{most}[A, B] = 1$ iff $\text{card}(A \cap B) > \text{card}(A - B)$
- Comparing blue and yellow dots directly.
Corresponding Procedures

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- Comparing number of target dots to the half of all dots.
- $\text{most}[A, B] = 1$ iff $\text{card}(A \cap B) > \text{card}(A - B)$
- Comparing blue and yellow dots directly.
- $\text{most}[A, B] = 1$ iff $\text{OneToOnePlus}(A \cap B, A - B)$
- Searching for 1-1 map.
Triggers
Experimental Questions

Question

Do people use OneToOnePlus strategy?
Experimental Questions

Question
Do people use OneToOnePlus strategy?

Question
Do people use ANS to judge truth-value?

Dehaene & Cohen, Cultural Recycling of Cortical Maps, Neuron, 2007

Pietroski et al., The Meaning of ‘Most’: semantics, Numerosity, and Psychology, Mind and Language, 1999
Pietroski’s et al. Experimental Design

Most of the dots are yellow.

Test sentence: 7000ms

Trial 1:
- Stimulus: 200ms
- Black screen: 4000ms

Trial 2:
- Stimulus: 200ms
- Black screen: 4000ms

Trial 3:
- Stimulus: 200ms
- Black screen: 4000ms

...
12 subjects
Experimental Design cont.

- 12 subjects
- 360 trials each
Experimental Design cont.

- 12 subjects
- 360 trials each
- 2 color dots: blue and yellow
Experimental Design cont.

- 12 subjects
- 360 trials each
- 2 color dots: blue and yellow
- The number of dots in each color: 5–17
Experimental Design cont.

- 12 subjects
- 360 trials each
- 2 color dots: blue and yellow
- The number of dots in each color: 5–17
- 9 bins: 1:2, 2:3, 3:4, 4:5, . . . , 9:10
Experimental Design cont.

- 12 subjects
- 360 trials each
- 2 color dots: blue and yellow
- The number of dots in each color: 5–17
- 9 bins: 1:2, 2:3, 3:4, 4:5, ..., 9:10
- Area-controlled vs. size-controlled
Experimental Design cont.

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- 10 trials in each bin for 4 conditions:
  - Scattered Random
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- 10 trials in each bin for 4 conditions:
  - Scattered Random
  - Scattered Pairs
  - Column Pairs Mixed
Experimental Design cont.

- 12 subjects
- 360 trials each
- 2 color dots: blue and yellow
- The number of dots in each color: 5–17
- 9 bins: 1:2, 2:3, 3:4, 4:5, ..., 9:10
- Area-controlled vs. size-controlled
- 10 trials in each bin for 4 conditions:
  - Scattered Random
  - Scattered Pairs
  - Column Pairs Mixed
  - Column Pairs Sorted
Trial Types

Scattered Random

Scattered Pairs

Column Pairs Mixed

Column Pairs Sorted
Results

- Participants did better with larger ratios.
- They did best on Column Pairs Sorted,
- but no significant differences among other trial types.
- No influence of size- or area-control.
Results

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Conclusion

*OneToOnePlus is out.*
Agreement with ANS

Observation

Agreement between the ANS and performance.
Agreement with ANS

Observation
Agreement between the ANS and performance.

Conclusion
Participants relied on ANS to evaluate ‘most’.
Question

*How is the cardinality of the non-blue set estimated?*

\[
\text{card}(\text{Dot} \cap \text{Yellow}) > \text{card}(\text{Dot} \cap \overline{\text{Yellow}})
\]

Lidz et al., Interface Transparency and the Psychosemantics of ‘most’, Natural Language Semantics, in press
2 Strategies: Selection

\[ \text{card}(Dot \cap Yellow) > \text{card}(Dot \cap Blue) + \text{card}(Dot \cap Red) + \ldots + \text{card}(Dot \cap Green) \]
2 Strategies: Selection

\[ \text{card}(\text{Dot} \cap \text{Yellow}) > \text{card}(\text{Dot} \cap \text{Blue}) + \text{card}(\text{Dot} \cap \text{Red}) + \ldots + \text{card}(\text{Dot} \cap \text{Green}) \]
2 Strategies: Subtraction

\[ \text{card}(\text{Dot} \cap \text{Yellow}) > \text{card}(\text{Dot}) - \text{card}(\text{Dot} \cap \text{Yellow}) \]
2 Strategies: Subtraction

\[ \text{card}(\text{Dot} \cap \text{Yellow}) > \text{card}(\text{Dot}) - \text{card}(\text{Dot} \cap \text{Yellow}) \]
Comparing 2 Strategies

- For 2 color screens selection is easier.
- For multi-color screens subtraction might be more optimal.

Question
Is one of them the default strategy?
Restrictions of Our Visual System

- We can generate estimates only for up to 3 sets: the total set of dots and 2 color subsets.
- So, selection might be out as a non-universal strategy.
Restrictions of Our Visual System

- We can generate estimates only for up to 3 sets: the total set of dots and 2 color subsets.
- So, selection might be out as a non-universal strategy.

Halberda et al., Multiple Spatially Overlapping Sets Can be Enumerated in Parallel, Psychological Science, 2006. DEMO
10 subjects;  
450 trials each;  
500 ms displays with 1-35 dots in 1-6 colors;  
probe before or after diverged from 3 colors on.
Lidz’s et al. Experimental Design

- **Test Sentence**: 7000 ms
  - Most of the dots are yellow.

- **Trial 1**: 4000 ms
  - **Stimulus**: 200 ms
  - **Black Screen**: yes/no

- **Trial 2**: 4000 ms
  - **Stimulus**: 200 ms
  - **Black Screen**: yes/no

- **Trial 3**: 4000 ms
  - **Stimulus**: 200 ms
Findings

Observation
There was no difference in accuracy as the function of number of colors on the display, but only as the function of the ratio.

Observation
Accuracy was not higher on 2 color screens.

Conclusion
Subtraction was always used.
Canonical Meaning of ‘most’

\[
\text{most}[A, B] = 1 \iff \text{card}(A \cap B) > \text{card}(A) - \text{card}(A \cap B)
\]
Discussion

Observation

For 2 colors selection is more efficient than subtraction but the later was nevertheless used.

…our data support Interface Transparency Thesis (ITT), according to which speakers exhibit a bias towards the verification procedures provided by canonical specification of truth-conditions. (Lidz et al., 2010)
Meaning as a Collection of Procedures

Subtractions might be the default strategy only when:

▶ Brief flash;
▶ The same sentence over and over again;

**Question**

*What are the other alternatives?*
Outline

Number Sense
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Approximating ‘most’

Superlative Meaning of ‘most’

What About Counting?
Tomaszewicz’s Study

Most 1: \( \text{większość, ‘majority’} \)
\[ \text{więk-} -sz- -ość \]
‘many/great’ adjectival comparative ‘-er/more’ nominalizer

Most 2: \( \text{najwięcej, ‘largest subset / the most’} \)
\[ \text{naj-} -więć- -ej \]
adverbial superlative ‘-est/most’ ‘many/great’ adverbial comparat. ‘-er/more’

\[ \text{card}(Dot \cap \text{Yellow}) > \text{card}(Dot \cap \text{Blue}) \& \]
\[ \& \quad \text{card}(Dot \cap \text{Yellow}) > \text{card}(Dot \cap \text{Red}) \& \]
\[ \ldots \& \quad \text{card}(Dot \cap \text{Yellow}) > \text{card}(Dot \cap \text{Green}) \]

(Stepwise Selection)
Experiments

- Design similar to Pietroski et al. 2009 and Lidz et al. 2010.
- 20 subjects on-line;
- 200ms;
- 2x180 displays (for Most 1 and Most 2);
- 2-5 colors (1-3 distractors).
Results

- For *Most 1* the result of Lidz et al. and of Pietroski et al. were replicated.
- *Most 1* = most.
Results

- For *Most 1* the result of Lidz et al. and of Pietroski et al. were replicated.
- *Most 1* = most.
- For *Most 2*: effect of ratio and number of colors.
- Stepwise selection.
- Superlative meaning.
Results

- For *Most 1* the result of Lidz et al. and of Pietroski et al. were replicated.
- *Most 1* = most.
- For *Most 2*: effect of ratio and number of colors.
- Stepwise selection.
- Superlative meaning.
- Different accuracy patterns for *Most 1* and *Most 2* in the same 2-color displays.
Discrepancy with Halberda et al. 2006?

**Question**

*So, can we attend more than 3 colors?*

**Hypothesis**

*2 different tasks:*

1. *enumeration of subsets and*
2. *recognizing the largest one.*
Question

Does most have a superlative reading in English?

Kotek et al, *Most Meanings are Superlative, to appear in Syntax Semantics*
Main Claim

Hypothesis

*Most is ambiguous between:*

1. *dominant proportional reading;*
2. *latent superlative reading.*

Can the speakers really access a superlative meaning?
Experiment 1: Covered Box

- A sentence is shown;
- with ‘more than half’ or ‘most’;
- and a picture:

| Blue: 3 ; Red: 6 ; Yellow: 8 | Blue: 8 ; Red: 4 ; Yellow: 5 | Blue: ? ; Red: ? ; Yellow: ? |
| (a) False picture          | (b) Superlative picture      | (c) Covered box              |

- ‘Exactly one picture matches the sentence. Choose.’
Experiment 1: Covered Box

- A sentence is shown;
- with ‘more than half’ or ‘most’;
- and a picture:

<table>
<thead>
<tr>
<th>Blue: 3; Red: 6; Yellow: 8</th>
<th>Blue: 8; Red: 4; Yellow: 5</th>
<th>Blue: ?; Red: ?; Yellow: ?</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) False picture</td>
<td>(b) Superlative picture</td>
<td>(c) Covered box</td>
</tr>
</tbody>
</table>

- ‘Exactly one picture matches the sentence. Choose.’

Results:

- Most: (c) 67.4%; (b) 32.6%.
- More than half: (c) 100%.
Meaning of ‘most’

most = \{ \text{approximation+subtraction, stepwise selection (superlative)}, \ldots \}
Outline

Number Sense

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Superlative Meaning of ‘most’

What About Counting?
Unchallenged Alternative

Why not:

\[ \text{most}[A, B] = 1 \iff \text{card}(A \cap B) > \frac{\text{card}(A)}{2} \]
Observation
Successful verification of ‘most’ in cases with two salient subsets is achieved at 3 years 7 months of age.

Observation
The comprehension is independent of knowledge of exact number words.

Halberda et al., The Development of ‘Most’ Comprehension and Its Potential Dependence on Counting Ability in Preschoolers, Language Learning and Development, 2008. DEMO
Question
What happens under a different experimental paradigm?

Hypothesis
\( \text{card}(A \cap B) > \frac{\text{card}(A)}{2} \) seems rather like ‘more than half’?

Question
Are the strategies for ‘most’ and ‘more than half’ distinct?
Hackl’s Self-paced Counting

(1) Most of the dots are blue.
(2) More than half of the dots are blue.

Screen 1

Screen 2

Screen 3

Screen 4

Screen 5

T F

Hackl, On the Grammar and Processing of Proportional Quantifiers, Natural Language Semantics, 2009
Results Reported: Equivalence

Success Rate

Total RTs "Most" / "More than half"
(Size Corrected)
Let’s Look Into The Verification Process

(1) Most of the dots are blue.
(2) More than half of the dots are blue.

Screen 1

Screen 2

Screen 3

Screen 4

Screen 5

T F
Results: Difference on 4 Screens

Most and More Than Half (Size Corrected)
\[ \Delta ts \text{ for screens p to 3} \]
Discussion

- Speakers treat expressions as equivalent;
- but they use different verification strategies.
Modified SPC

(1) Most of the dots are blue.
(2) More than half of the dots are blue.
Manipulating Distribution in SPC

Item Schema for Experiment 3a, b

- Exp. 3a
- Exp. 3b
- Exp. 1

Number of dots in target color vs. Screen Number.
Modified SPC

(1) Most of the dots are blue.
(2) More than half of the dots are blue.

Screen 1

Screen 2

Screen 3

Screen 4

Screen 5

Screen 6

Screen 7

T
F
Results

Early/Late  Most vs. More than half
Discussion

- ‘most’ is more sensitive to distributional asymmetries.
- It needs more time in L-condition.
Discussion

- ‘most’ is more sensitive to distributional asymmetries.
- It needs more time in L-condition.
- ‘more than half’ is almost unaffected.
Discussion

- ‘most’ is more sensitive to distributional asymmetries.
- It needs more time in L-condition.
- ‘more than half’ is almost unaffected.
Hackl’s Conclusions

▶ ‘most’ triggers ‘lead counting’:
  ▶ How much target color leads at every screen.
  ▶ Harder if it falls significantly behind.
Hackl’s Conclusions

- ‘most’ triggers ‘lead counting’:
  - How much target color leads at every screen.
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- but determining $\frac{A}{2}$ is insensitive to that.
Hackl’s Conclusions

▶ ‘most’ triggers ‘lead counting’:
  ▶ How much target color leads at every screen.
  ▶ Harder if it falls significantly behind.
▶ but determining \( \frac{A}{2} \) is insensitive to that.
▶ So:
  more than half\([A, B] = 1 \) iff \( \text{card}(A \cap B) > \frac{\text{card}(A)}{2} \)
Question

1. *It cries out for comparison with non-SPC situation?*
2. *Aren’t late and early in fact symmetric for ‘lead counting’?*
3. *Is the estimation of $\frac{A}{2}$ really insensitive?*
4. *What about carrying the memorized values over?*
Let’s us investigate the precise strategies closer!