1 Lambda’s and types

1.1 Lambda’s

Let us assume the following type declarations:
- $x, y, z$ are variables of type $e$
- $P, Q$ are variables of type $\langle e, t \rangle$
- $R$ is a variable of type $\langle e, \langle e, t \rangle \rangle$
- $M$ is a variable of type $\langle \langle e, t \rangle, \langle e, t \rangle \rangle$
- $m$ is a variable of type $t$
- $n$ is a variable of type $\langle e, t \rangle$

(a) Which of the following expressions are well-formed?
1. $\lambda x \lambda R(M(R(x) = Q))$
2. $(\lambda Q \lambda P(P = Q))(Q)(M(Q))$
3. $(\lambda n(n(\exists y(P(y) \land n(z = y))))(\lambda m \neg m))$

(b) Which of the preceding well-formed expressions can be simplified by $\lambda$- conversion. How?

(c) Which of the following expressions are equivalent:
1. $(\lambda x \forall z(\neg P(z) \leftrightarrow z = x))(z)$
2. $(\lambda x \forall y(P(y) \lor y = x))(z)$
3. $\neg P(z) \land \neg \exists y(\neg P(y) \land z \neq y)$
4. $(\lambda m (n(\exists y(n(P(y)) \land n(z = y))))(\lambda m \neg m))$

1.2 Types

- If $a$ and $b$ are expressions of type $e$, what is the type of $g$ in the following expression: $(g((g(b))(a)))(b)$? And what is the type of the whole expression?

- Is it possible to assign types to $\alpha, \beta$ and $\gamma$ such that both $(\alpha(\beta))(\gamma)$ and $(\alpha(\beta(\gamma)))$ are well-formed expressions?
2 Extending MG (forget about the intensional part here)

2.1 ... with three place verbs
Extend the grammar given in ‘A crash course in Montague Grammar’ with a three place verb like give, so that it generates sentences like Every girl gives Mary a book. Illustrate how your extension works by giving a full derivation and translation of the two readings of this sentence.

2.2 ... with only
Add a term modifier only to the fragment, that is, an expression of category $T/T$
What is the corresponding type?
Assign a translation to only such that it produces appropriate interpretations of sentences like Only John walks, and John loves only Mary (Again, think of what the desired result must be, and then reconstruct how you can obtain that, given that, e.g. John translates as $\lambda P.P(j)$.)

Does your translation of only still work if you apply it to a conjunction of proper names, as in Only [John and Mary]? If it doesn’t, sketch why.