

Quantifiers & Cognition

Lecture 2: Quantifiers and Approximation

Jakub Szymanik

Outline

Number Sense

Approximating 'most'

What about counting?

Outline

Number Sense

Approximating 'most'

What about counting?

Departure point

Observation

Quantifiers are associated with truth-conditions.

Departure point

Observation

Quantifiers are associated with truth-conditions.

Observation

Quantifiers are associated with verification strategies.

Number knowledge

Observation

Quantifiers embed number knowledge in language.

Number knowledge

Observation

Quantifiers embed number knowledge in language.

Question

What are the characteristics of that ability in humans?

Number Sense

Definition

An intuitive understanding of numbers, their magnitude, relationships, and how they are affected by arithmetical operations.



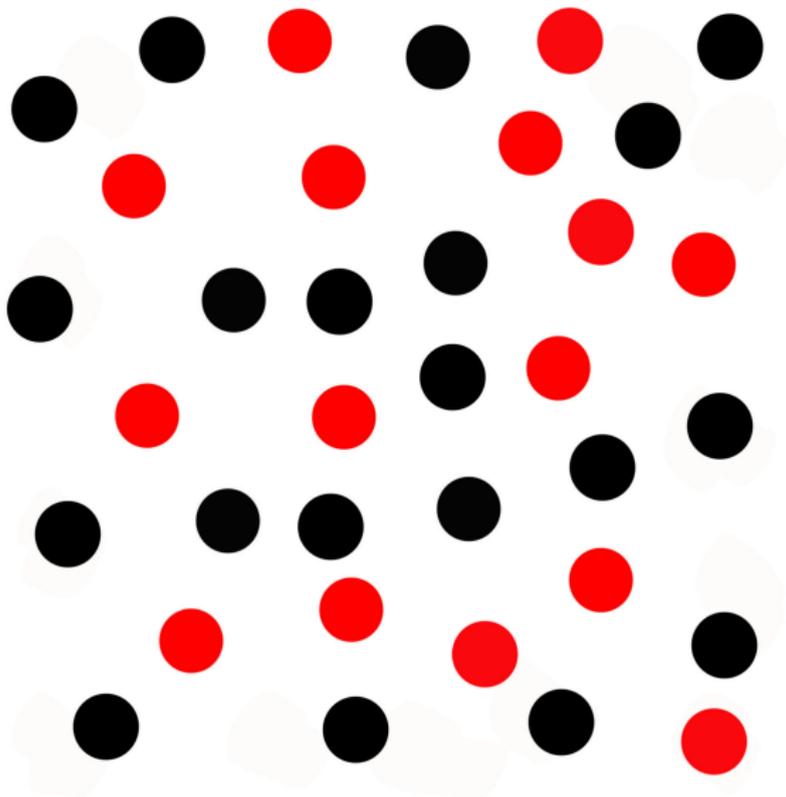
Dehaene, *The Number Sense: How the Mind Creates Mathematics*, 1999

Test your approximate number sense

Test

Approximate Number Sense

Approximate Number Sense



ANS cont.

- ▶ common to many nonverbal animals;

ANS cont.

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ANS cont.

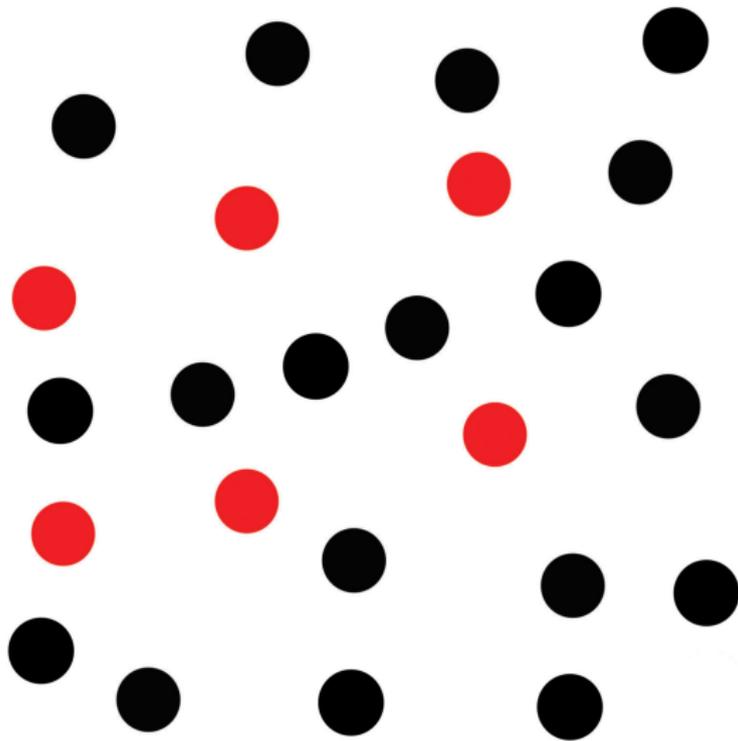
- ▶ common to many nonverbal animals;
- ▶ an evolutionary ancient cognitive resource;
- ▶ generates representations of numerosity;
- ▶ across multiple modalities;
- ▶ doesn't require explicit training;

ANS cont.

- ▶ common to many nonverbal animals;
- ▶ an evolutionary ancient cognitive resource;
- ▶ generates representations of numerosity;
- ▶ across multiple modalities;
- ▶ doesn't require explicit training;
- ▶ but works with certain limits.

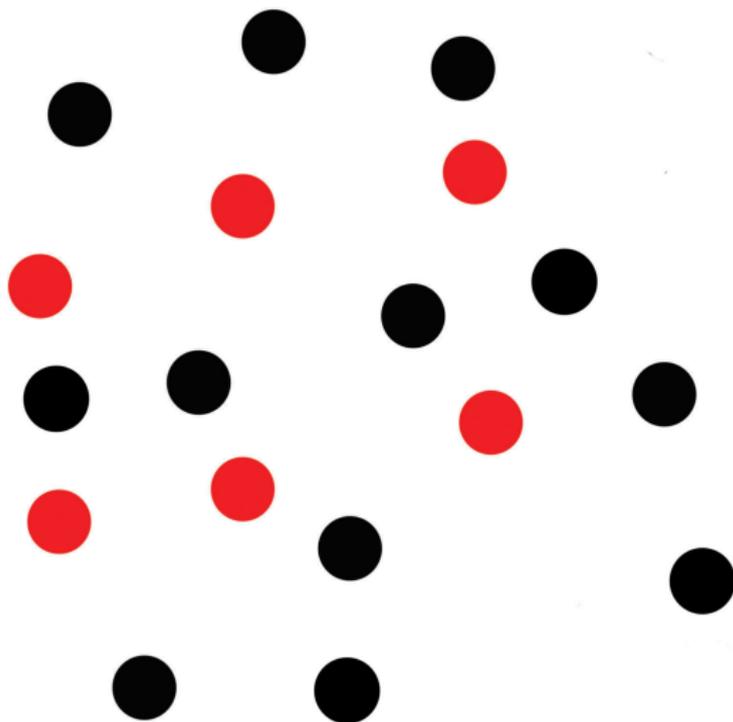
Distance effect

Distance effect



Distance effect

Distance effect



Distance effect cont.

Observation

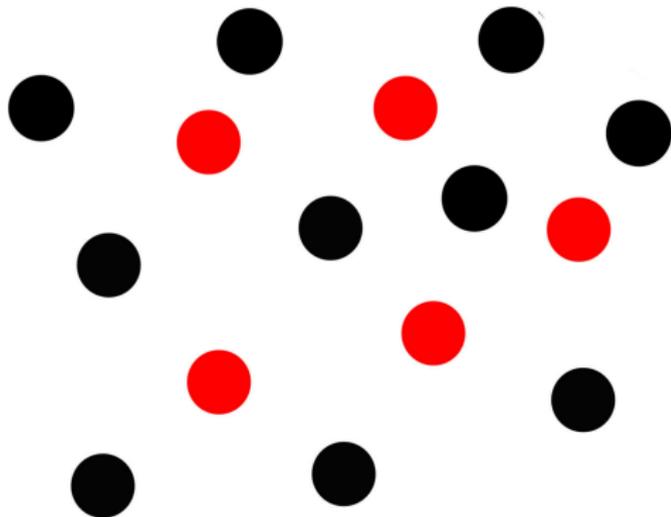
There is a systematic, monotonous decrease in numerosity discrimination performance as the numerical distance between the numbers decreases.

Example

From 6 vs 18 to 6 vs 12.

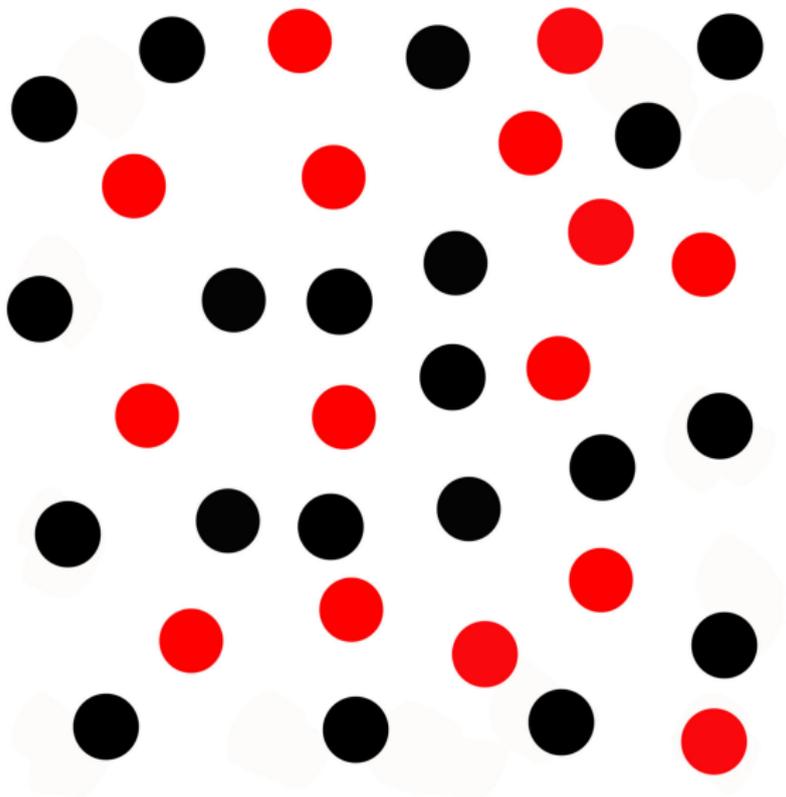
Size-effect

Size-effect



Size-effect

Size-effect



Size-effect cont.

Observation

For equal numerical distance, performance also decreases with increasing number size.

Size-effect cont.

Observation

For equal numerical distance, performance also decreases with increasing number size.

Example

5 things are detectable different from 10 then 15 from 20.

Weber's Law

Weber's Law

Discriminability depends on the ratio of relevant representational values.

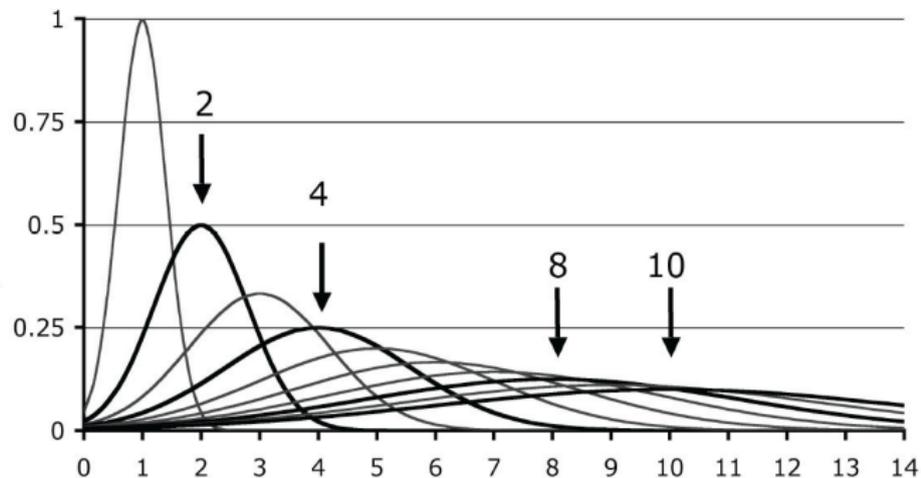
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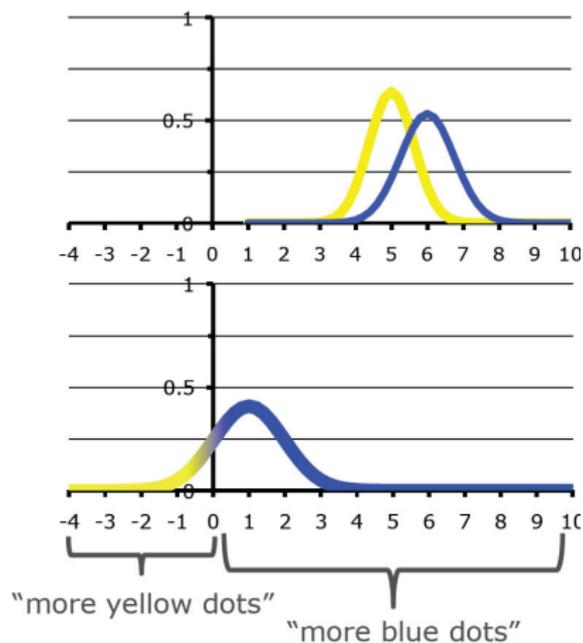
$$WR = \frac{\text{larger set}}{\text{smaller set}}$$

Mental number line



Individual differences (Weber fraction)

The more overlap, the poorer the discriminability.



Development of ANS

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- ▶ But 6 months infants only 1:2.

ANS and counting

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ANS and counting

- ▶ By 5yrs ANS is mapped onto discrete number words.
- ▶ It is activated anytime we deal with numbers.
- ▶ Distance and size effect for Arabic numerals!

Outline

Number Sense

Approximating 'most'

What about counting?

Meaning of 'most'

$$\text{most}[A, B] = 1 \text{ iff } \text{card}(A \cap B) > \frac{\text{card}(A)}{2}$$

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Meaning of 'most' cont.

$$\text{most}[A, B] = 1 \text{ iff } \text{OneToOnePlus}(A \cap B, A - B)$$

Definition

$\text{OneToOnePlus}(X, Y) \iff \exists X' \subset X$ s.t. there is a one-to-one function between X' and Y but not X and Y .

Corresponding procedures

They suggest different strategies:

Corresponding procedures

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- ▶ Comparing number of target dots to the half of all dots.

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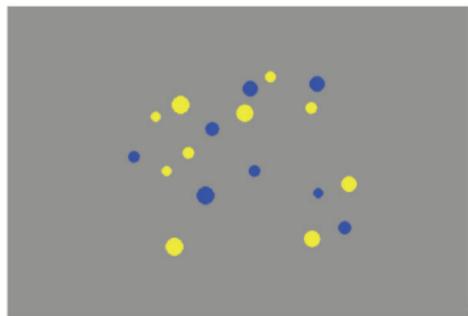
- ▶ Comparing number of target dots to the half of all dots.
- ▶ Comparing blue and yellow dots directly.

Corresponding procedures

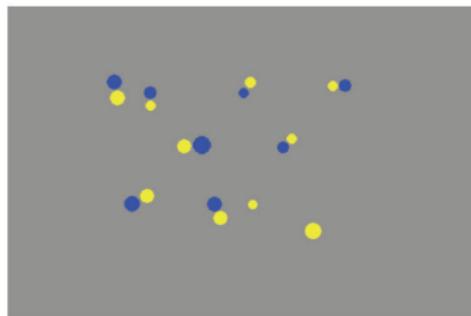
They suggest different strategies:

- ▶ Comparing number of target dots to the half of all dots.
- ▶ Comparing blue and yellow dots directly.
- ▶ Searching for 1-1 map.

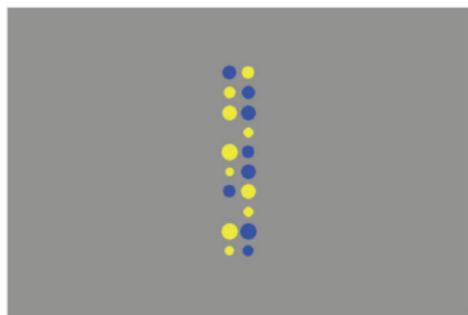
Triggers



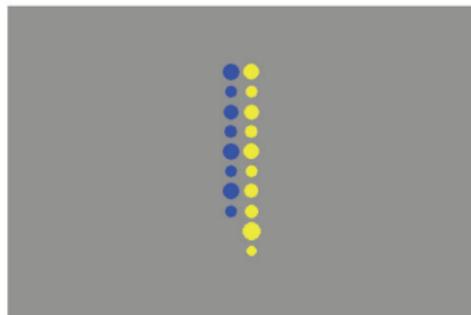
a



b



c



d

Experimental questions

Question

Do people use OneToOnePlus strategy?

Experimental questions

Question

Do people use OneToOnePlus strategy?

Question

Do people use ANS to judge truth-value?

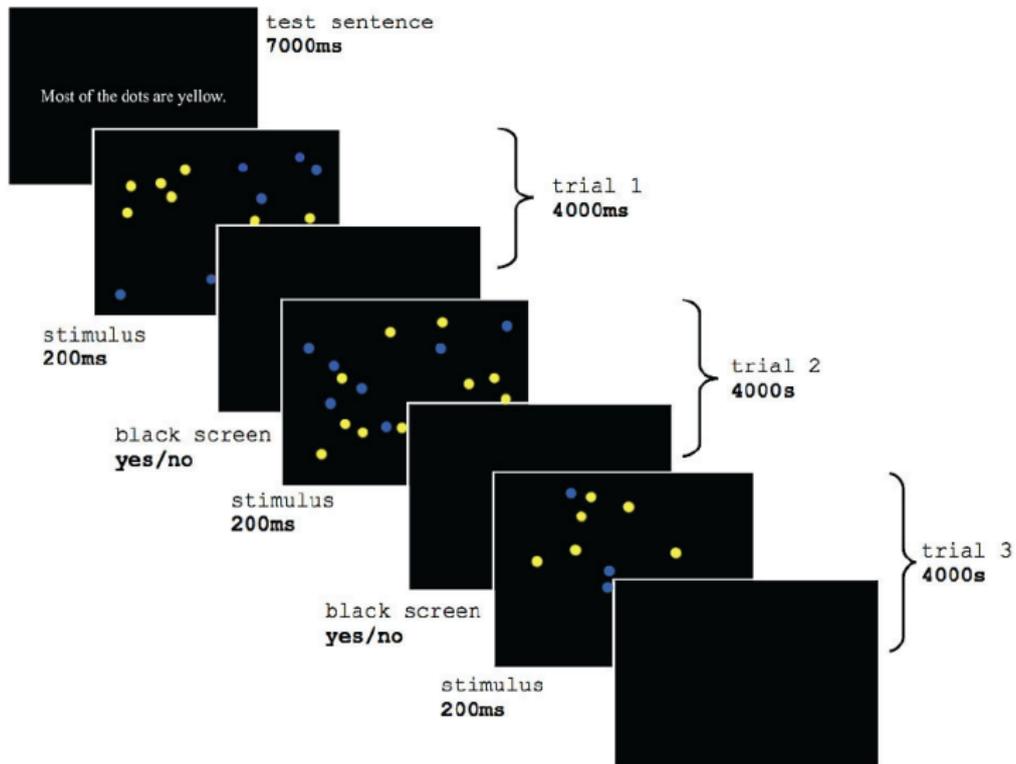


Dehaene & Cohen, Cultural recycling of cortical maps, *Neuron*, 2007



Pietroski et al., The Meaning of 'Most': semantics, numerosity, and psychology, *Mind and Language*, 1999

Experimental design



Experimental design cont.

- ▶ 12 subjects

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- ▶ 12 subjects
- ▶ 360 trials each

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Experimental design cont.

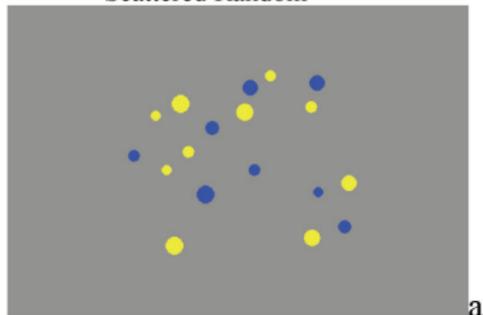
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Experimental design cont.

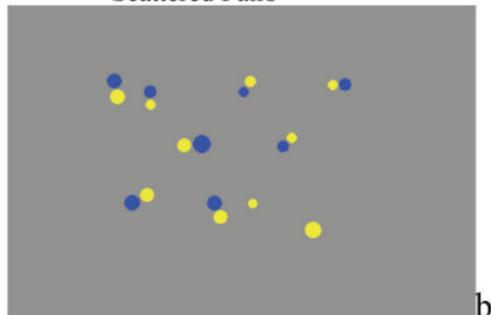
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 - ▶ Column Pairs Mixed
 - ▶ Column Pairs Sorted
- ▶ Size-controlled vs. area-controlled

Trial types

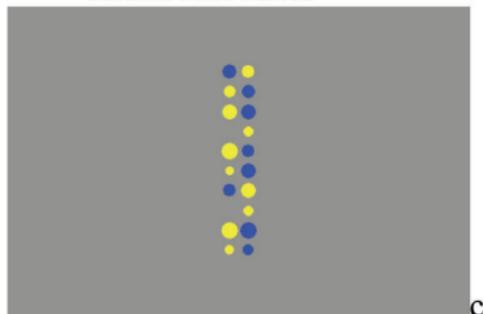
Scattered Random



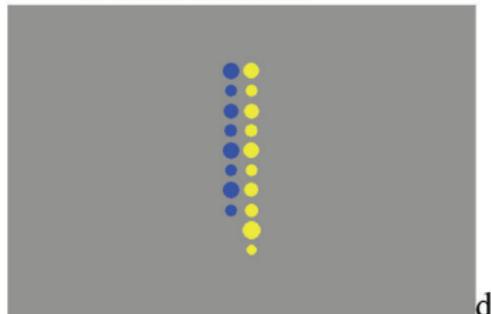
Scattered Pairs



Column Pairs Mixed



Column Pairs Sorted



Results

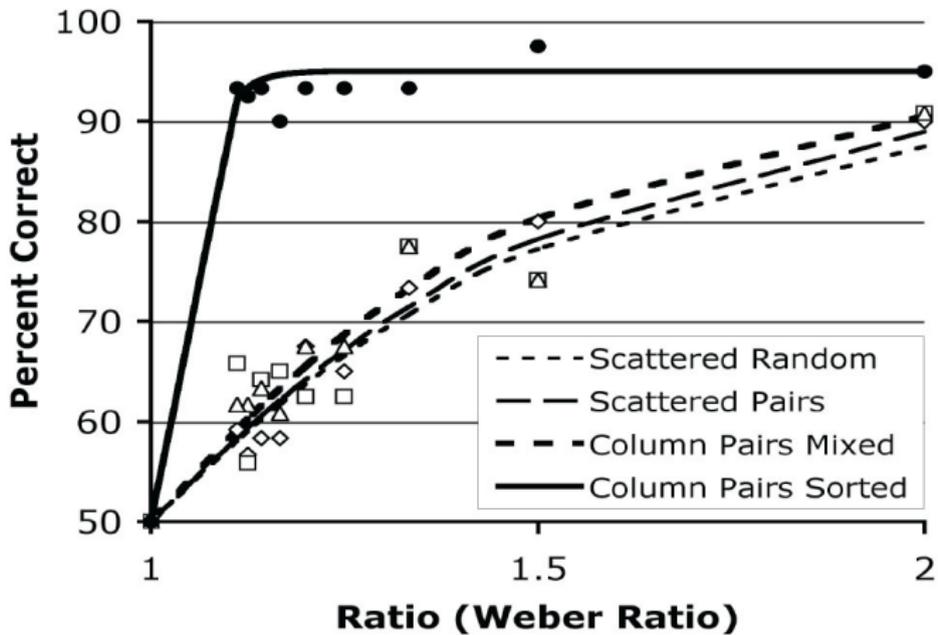
- ▶ Participants did better with easier ratios.
- ▶ They did best on Column Pairs Sorted,
- ▶ but no significant differences among other trial types.
- ▶ No influence of size- or area-control.

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Conclusion

OneToOnePlus is out.



Agreement with ANS

Observation

Author found out agreement between the psychophysical model of the ANS and participants' performance.

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Participants relied on ANS to evaluate 'most'.

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Participants relied on ANS to evaluate 'most'.

Question

Re-cycling of cortical maps?

Follow-up question

Question

How is the cardinality of non-blue set estimated?



Lidz et al., Interface Transparency and the Psychosemantics of 'most', Natural Language Semantics, in press

2 strategies

- ▶ Selection:

$$\text{card}((A \cap C) \cup \dots \cup (A \cup Z))$$

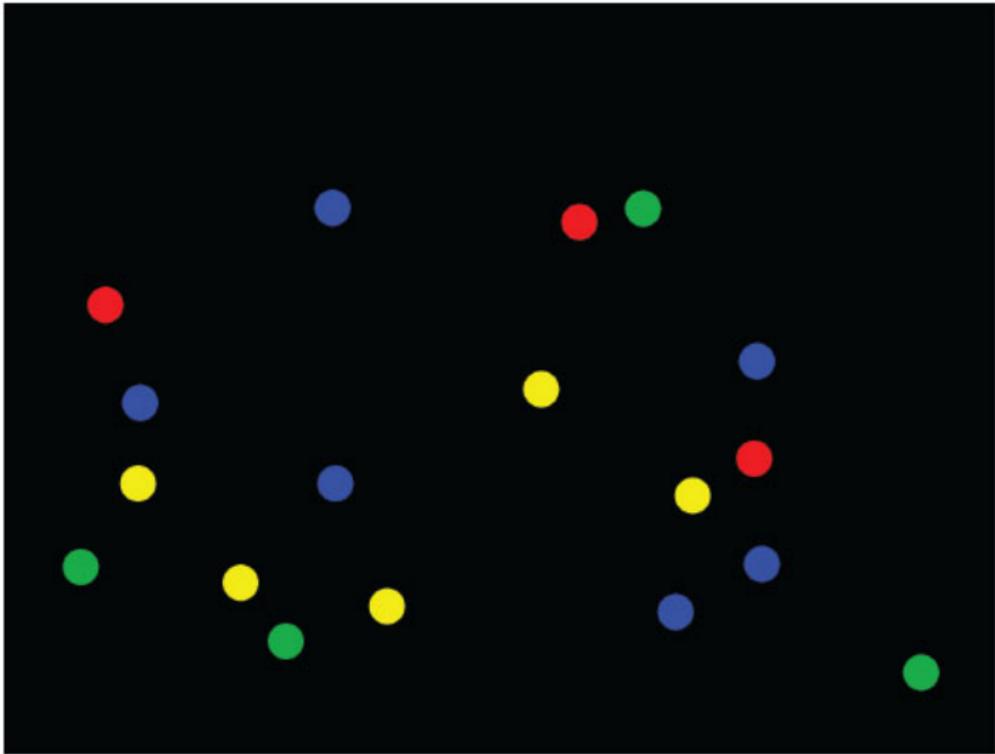
2 strategies

- ▶ Selection:

$$\text{card}((A \cap C) \cup \dots \cup (A \cup Z))$$

- ▶ Subtraction:

$$\text{card}(A) - \text{card}(A \cap B)$$

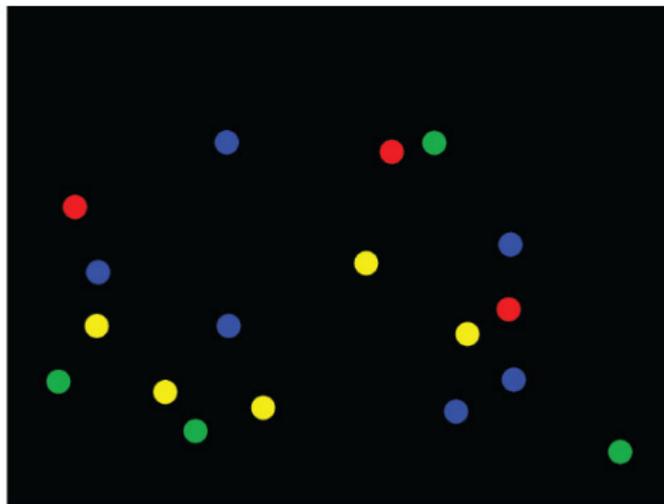


Multi-sets

- ▶ We can generate estimates only for up to 3 sets.
- ▶ So, selection is out as a universal strategy.

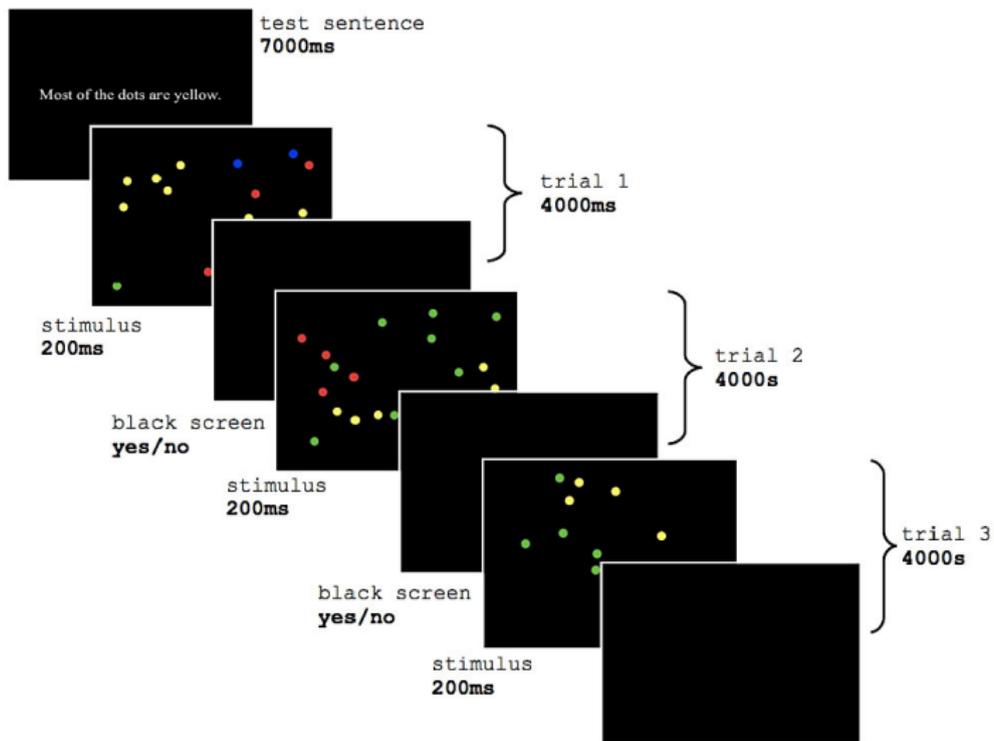
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Halberda et al., Multiple Spatially Overlapping Sets Can be Enumerated in Parallel, Psychological Science, 2006

Design



Findings

Observation

There was no difference in accuracy as the function of number of colors on the display, but only as the function of the ratio.

Conclusion

Subtraction was always used.

Canonical meaning of 'most'

$$\text{most}[A, B] = 1 \text{ iff } \text{card}(A \cap B) > \text{card}(A) - \text{card}(A \cap B)$$

Discussion

Observation

For 2 colors selection is more efficient than subtraction but the later was nevertheless used.

... our data support Interface Transparency Thesis (ITT), according to which speakers exhibit a bias towards the verification procedures provided by canonical specification of truth-conditions. (Lidz et al., 2010)

Outline

Number Sense

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What about counting?

Unchallenged alternative

Why not:

$$\text{most}[A, B] = 1 \text{ iff } \text{card}(A \cap B) > \frac{\text{card}(A)}{2}?$$

Children and 'most'

Observation

Successful 'most' comprehension in cases with two salient subsets is achieved at 3 years 7 months of age.

Observation

The comprehension is independent of knowledge of exact number words.



Halberda et al., The Development of 'Most' Comprehension and Its Potential Dependence on Counting Ability in Preschoolers, Language Learning and Development, 2008

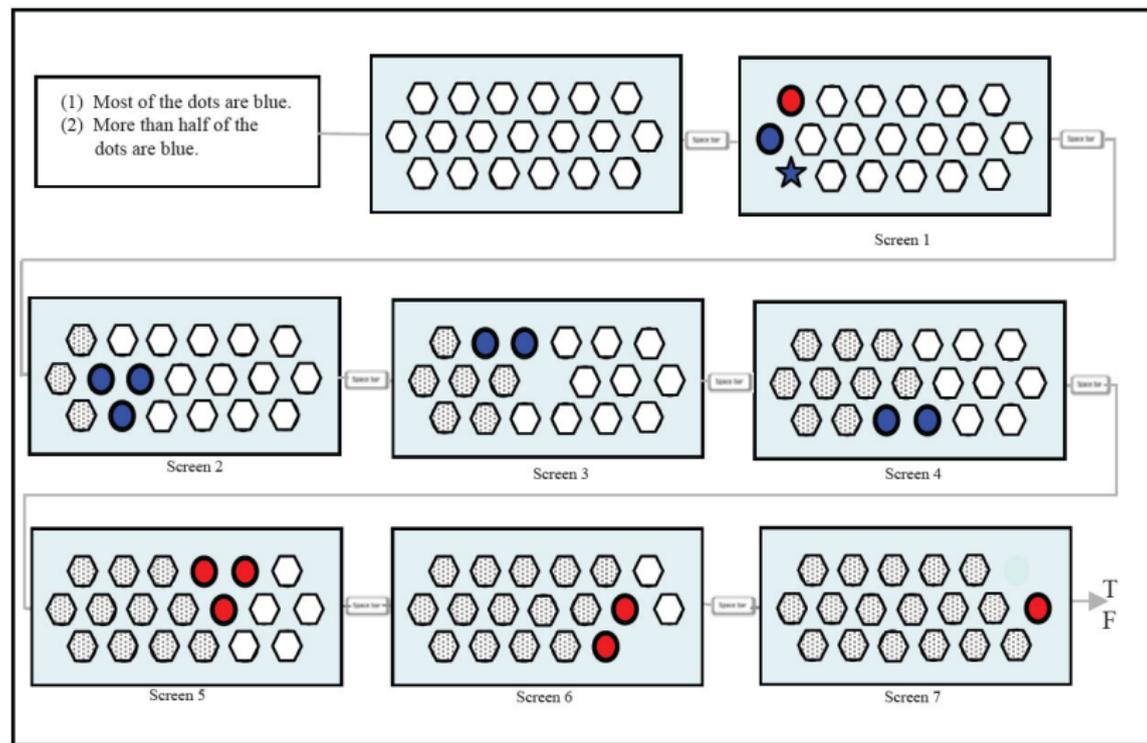
Translating ANS into numbers?

Our estimate of a Weber fraction of approximately .3 for three trial types suggests that participants may be translating the representation of the ANS into whole number values via some numeralizing waystation before evaluating 'most'-sentences. For example, when shown an array of 12 blue and 16 yellow dots, these subset may activate corresponding ANS representation of numerosity; and these values may be translated into cardinal-number estimates, like 'twelve' and 'sixteen', for purposes of evaluation. Further work is needed to determine if this is the case. (Pietroski et al., 2009)

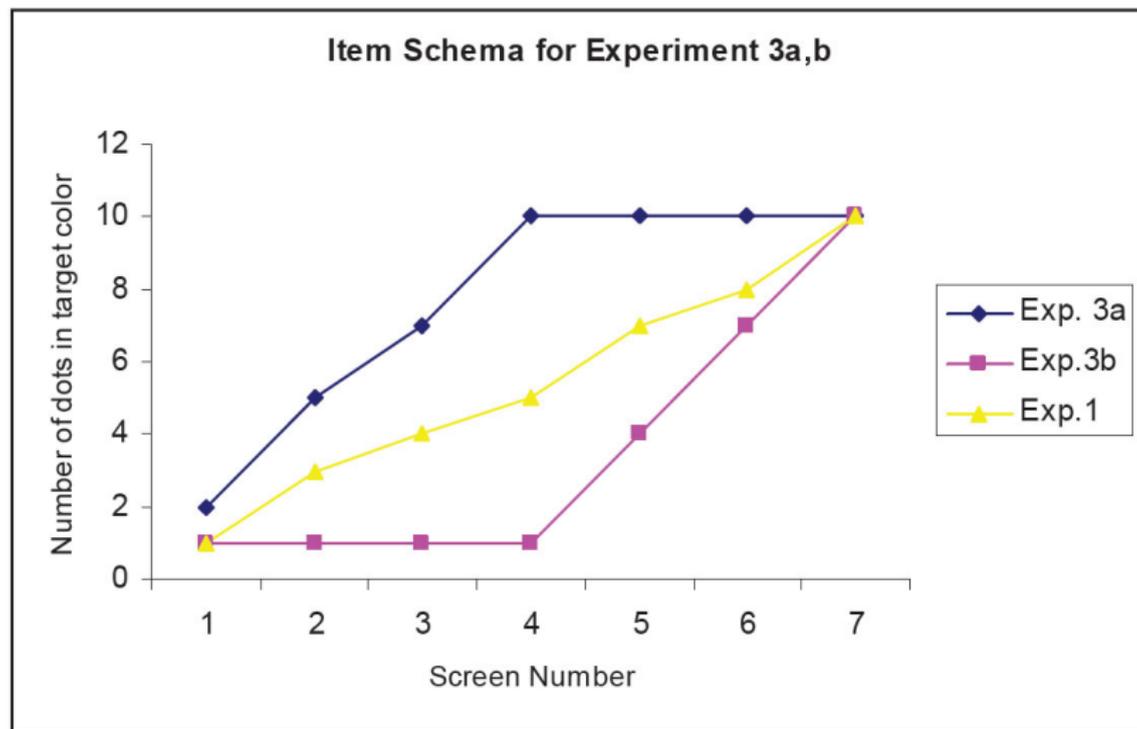
What about strategies for 'more than half'?

- ▶ Hackl suggests that they are different than for 'most'.
- ▶ So maybe we count here?

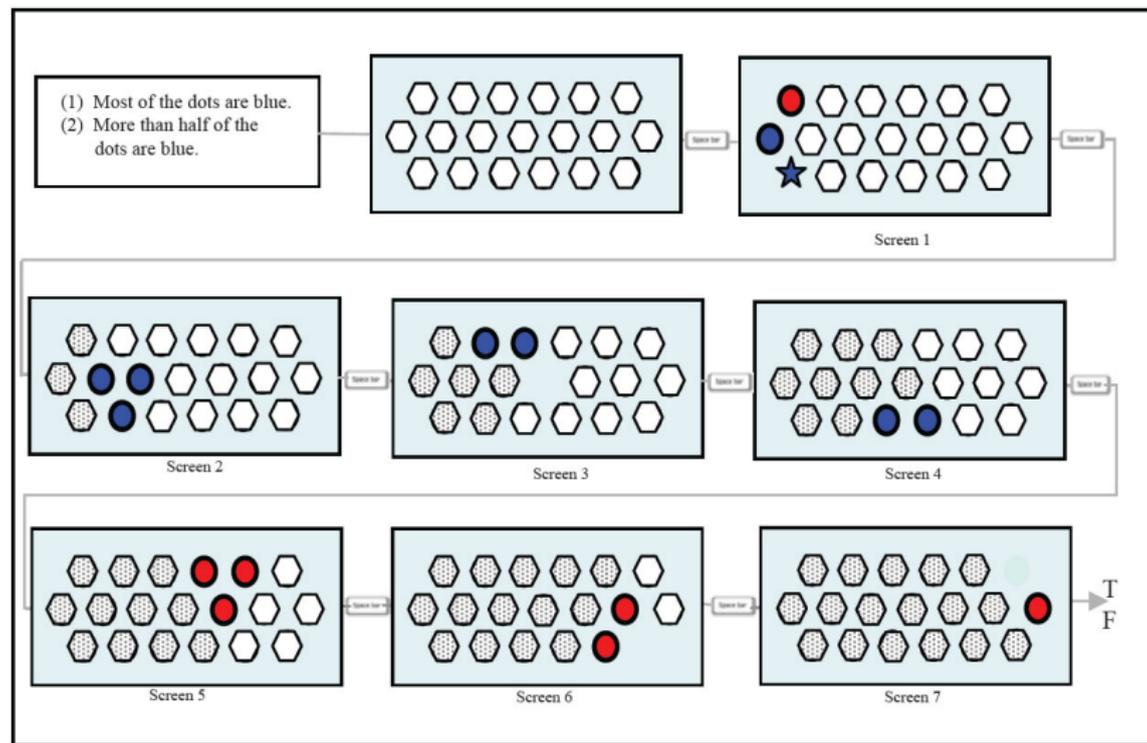
Modified SPC



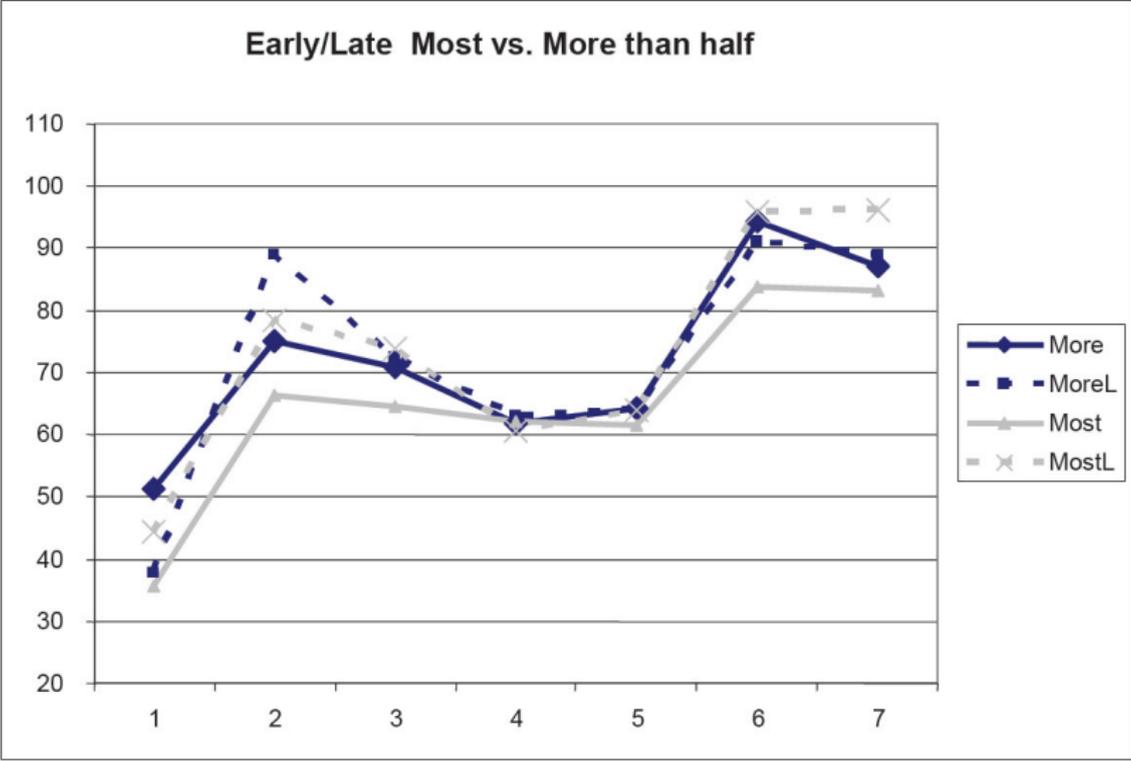
Manipulating distribution in SPC



Modified SPC



Results



Discussion

- ▶ 'most' is sensitive to distributional asymmetries.
- ▶ It needs more time in L-condition.

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Hackl's conclusions

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Hackl's conclusions

- ▶ 'most' triggers 'lead counting':
 - ▶ How much target color leads at every screen.
 - ▶ Harder if it falls significantly behind.
- ▶ but determining $\frac{A}{2}$ is insensitive to that.
- ▶ So:
more than half $[A, B] = 1$ iff $\text{card}(A \cap B) > \frac{\text{card}(A)}{2}$

However,...

Question

1. *It cries out for comparison with normal situation?*
2. *Aren't late and early in fact symmetric for 'lead counting'?*
3. *Is the estimation of $\frac{A}{2}$ really insensitive?*
4. *What about carrying the memorized values over?*

Let's us investigate the precise strategies closer!