

MSc AI and HMC thesis project:  
**Modeling Quantifier Verification**

**Background.** Animals are able to count and represent quantities, but reasoning with linguistic expressions of (relative) quantities (known as quantifiers) seems a uniquely human ability. We can understand, for example, sentences such as 'Most linguists are logicians', 'Less than half of the cognitive neuroscientists are computer scientists', and 'At least 3 of the applicants are psychologists.' We can also assess the conditions that make such sentences true or false. While the cognitive bases of counting and quantity representations have been extensively studied (see, e.g., Dehaene 1999), the cognitive processing of linguistically expressed quantities is far from being understood. Quantifier expressions occur whenever we describe the world, and communicate about it. Generalized quantifier theory studies the possible meanings and the inferential power of quantifier expressions by logical means. The classical version was developed in the 1980s, at the interface of linguistics, mathematics and philosophy. Until now, advances in "classical" generalized quantifier theory mainly focused on definability questions and their applications to linguistics (see Peters and Westerståhl 2006 for an overview). However, generalized quantifiers have been also studied from psychological perspective (see, e.g., Moxey and Sanford 1993; Clark 1976).

**Project.** McMillan et al. (2005, 2006) examined the pattern of neuroanatomical recruitment while subjects were judging the truth-value of statements containing natural language quantifiers. The authors considered two standard types of quantifiers: definable in first-order logic (e.g., 'all', 'some', 'at least 3'), and not definable in first-order logic, so-called higher-order quantifiers, (e.g., 'an even number of', 'more than half'). They concluded that all quantifiers recruit the right inferior parietal cortex, which is associated with numerosity, but only higher-order quantifiers recruit the prefrontal cortex, which is associated with executive resources, like working memory.

From a computational point of view to recognize first-order quantifiers we only need computability models that do not use any form of working memory. Intuitively, to check whether 'Every sentence in this proposal is correct' we do not have to remember anything. It suffices to read the sentences from this article one by one. If we find an incorrect one, then we know that the statement is false. Otherwise, if we read the entire proposal without finding any incorrect sentence, then the statement is true. We can proceed in a similar way for other first-order quantifiers. It was proved by Van Benthem (1986) that such simple devices as finite-state automata can compute first-order quantifiers. However, for recognizing some proportional higher-order quantifiers, like 'more than half' we need computability models making use of unbounded working memory. Intuitively, to check whether 'More than half of the sentences in this proposal are correct' we must identify the number of correct sentences and hold it in working memory to compare with the number of incorrect sentences. Mathematically speaking, such an algorithm cannot be realized by a finite-state automaton, one needs a push-down machine.

Following above distinctions Szymanik and Zająkowski (2010, 2011) studied this hypothesis experimentally by comparing the processing of various classes of quantifiers with respect to their computational complexity. The authors concluded that proportional quantifiers are the hardest to verify and engage working memory to the highest degree and the difficulty of other quantifiers is well approximated by the computational model, e.g., by the number of states in the corresponding finite-automaton.

The aim of the project is to build a psychologically and neurally plausible theory of quantifier meaning and maybe compare it with other proposals, like Johnson-Laird's mental models (2006). There are many questions about the correspondence between the formal model of quantifier verification and the cognitive resources the subjects need to use in order to solve the task. For example, is it rather 'storage' or 'processing/comparison' that makes 'more than half' exceptionally difficult? This and similar questions could be answered by embedding the formal model into computational cognitive architecture, like ACT-R. For instance, from a modeling perspective, 'more than half' requires maintaining two values as opposed to one in the 'more than k' case. This may lead to an overextension of immediate memory, and therefore to a slower response. Building an ACT-R model can lead to a new experimental predictions that can be consequently tested. Moreover, none of the empirical research so far has looked into actual strategies the subjects are

applying in order to verify quantifier sentences. Eye-tracking study could fill the gap and provide additional data whether the models of quantifier verification postulate psychologically plausible strategies.

**Student.** We are looking for one or two MSc student in HMC or AI familiar with computational cognitive modeling and/or experimentation with adult subjects and statistical methods. Some familiarity with mathematical linguistics (automata theory, formal semantics) is useful as well. The project involves developing and ACT-R model of quantifier verification and/or eye-tracking study. This research project will be jointly supervised by Niels Taatgen and Jakub Szymanik (both Multi-Agent Systems Group here at ALICE).

## References

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